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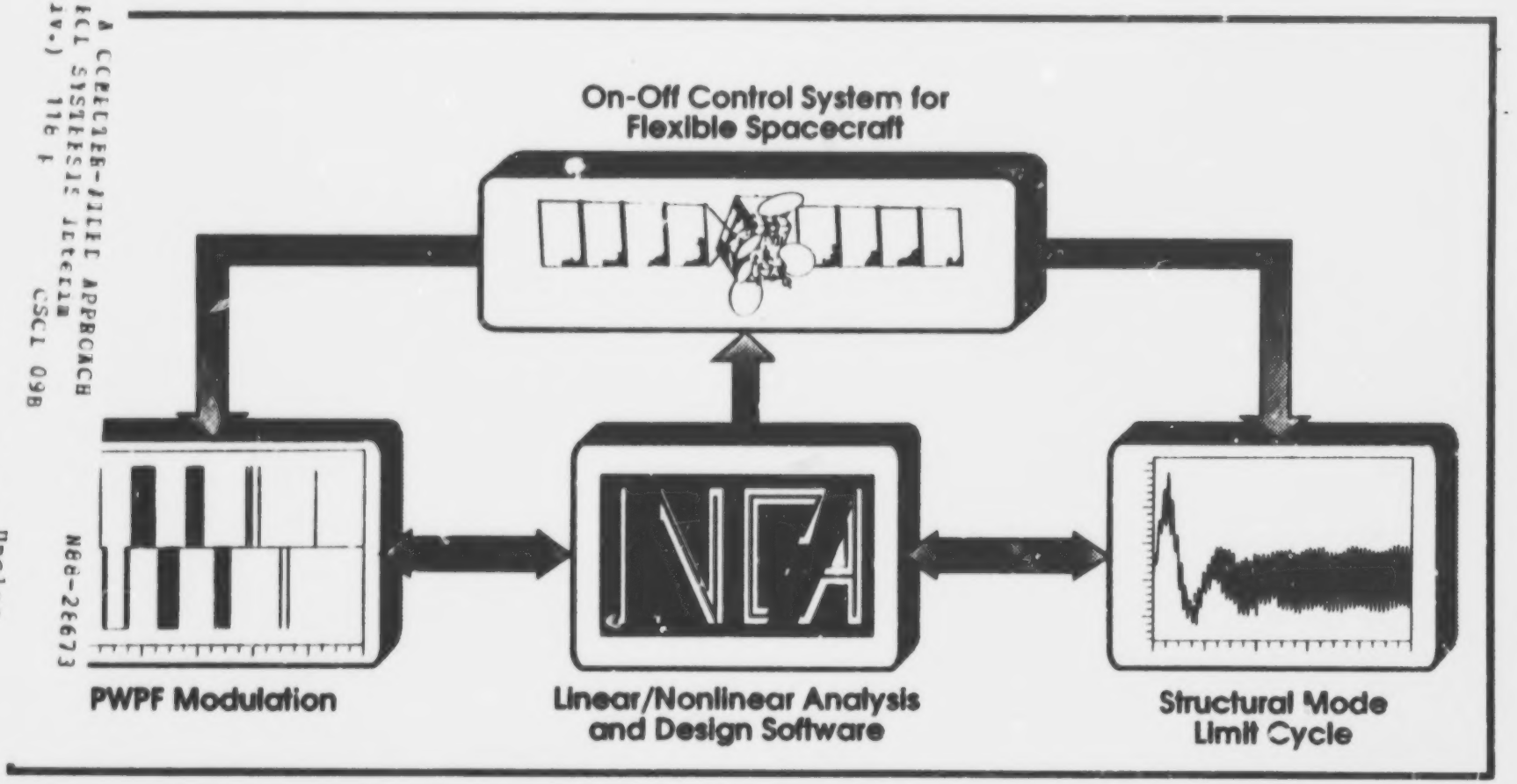
A Computer-Aided Approach to Nonlinear Control Synthesis

Interim Report
July 19, 1988
Contract NAS8-36224



The University of Texas at Austin / Dept. of Aerospace Engineering and Engineering Mechanics

Contract NAS8-36224

A Computer-Aided Approach to Nonlinear Control Synthesis

Interim Review

July 19, 1988

Prepared by

Bong Wie and Tobin Anthorv
Dept. of Aerospace Engineering
The University of Texas at Austin
Austin, Texas 78712

Presented to

NASA Marshall Space Flight Center
Huntsville, Alabama

A Computer-Aided Approach to Nonlinear Control Synthesis

An Interim Report (July 19,1988)

Contract Number: NAS8-36224

Contractor: Dept. of Aerospace Engineering
and Engineering Mechanics
The University of Texas at Austin
Austin, Texas 78712

Effective Date of Contract: June 24, 1987
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Principal Investigator: Dr. Bong Wie (512) 471-5322
Technical Contributors*: Tobin C. Anthony
C.-H. Jen
Dr. C.-H. Chuang

Prepared for NASA Marshall Space Flight Center, Huntsville, Alabama 35812

Technical Monitor: Stanley Carroll, ED13
NASA Marshall Space Flight Center
Huntsville, AL 35812
(205) 544-1472

*The adaptations accomplished in this project involve inclusion of the describing function capability into version 3.0 of the Interactive Controls Analysis (INCA) program. Members of the project are grateful for the technical assistance given by Dr. John Downing and Mr. Frank Bauer of NASA-Goddard Space Flight Center who offered help and ideas for the framework of the describing function modifications.

INTRODUCTION AND SUMMARY

The major objective of this project is to develop a computer-aided approach to nonlinear stability analysis and nonlinear control system design. This goal is to be obtained by refining the describing function method as a synthesis tool for nonlinear control design. This interim report outlines the approach by this study to meet these goals including an introduction to the INteractive Controls Analysis (INCA) program which was instrumental in meeting these study objectives. A single-input describing function (SIDF) design methodology was developed in this study; coupled with the software constructed in this study, the results of this project provide a comprehensive tool for design and integration of nonlinear control systems.

Report Outline

The report consists of several sections in viewgraph format, a table of contents appears throughout the report to separate the different sections. In the first section, an introduction to the project is found; task information as well as a list of the project accomplishments, including an example of INCA's new describing function capability, are given. Next, a history of the INCA program is discussed as well as a brief explanation of its capabilities. The following section details the implementation of the SIDF analysis capability into INCA. The types of nonlinear devices modelled are described and a brief description of the theory behind the describing function method is given. The consequent section details the nonlinear control synthesis methodology for a flexible spacecraft; steps for developing the compensation for a nonlinear control system with flexible modes are presented. This methodology is used in the modelling of an actual flight-proven attitude control system, INTELSAT 5; this spacecraft employs the pulse-width pulse-frequency (PWPF) modulator which is one of the types of nonlinear controllers modelled in INCA. Five different types of linear compensation are substituted for the actual compensation; the describing function representation as well as the numerical simulation of each sample case is presented. The

next section details the plan for the second year of the contract; this phase of the contract involves much of the same analysis as this past year except the emphasis will be on dual-input describing functions. Outlines for possible INCA enhancements are then presented followed by a summary and conclusion of this phase of the contract. The appendix contains supplementary linear analyses of the INTELSAT 5 spacecraft control system presented earlier in the report.

Summary

The presence of nonlinear devices in a linear system often inhibits linear analysis of that system. Nonlinearities discussed in this report come in two forms: energy-storage and non-energy storage type. Non-energy storage systems are often representations of nonlinear effects which are inherent in the system such as deadzones, limiters, and Schmitt Triggers. Describing functions of the energy-storage type include such devices as the pulse-width pulse-frequency (PWPF) modulator, the derived-rate modulator, and the pulse-width modulator. These types of nonlinear controllers are used to modulate the reaction jets of spacecraft attitude control systems and are often referred to as pulse modulators.

The describing function method is often used to predict structural-mode limit cycling which may occur as a result of control system/structural interaction. By plotting the describing function of a nonlinearity with the corresponding frequency response plot, the designer can obtain information about the stability of the system with respect to structural limit cycling. For example, the intersection of the two curves indicates the possibility of a nonlinear instability; the probability of limit cycle occurring is based on the type of input to the system. It is important to note that for energy-storage type nonlinearities, the frequency of the intersection point must be the same on both curves. If there is no intersection between the curves, approximate phase and gain margins can be defined depending on the relative

locations of the two arcs. The concept of nonlinear stability margins is important in nonlinear control design methodology.

The describing function capabilities described in this report have been implemented in the INCA program. INCA is the result of a software development effort out of NASA Goddard Space Flight Center. Using INCA, the designer can execute linear controls analysis in a menu-driven, interactive graphics environment; root locus, frequency response, or time response analyses can be performed. Furthermore, with the use of a high-resolution graphics terminal, a user can query the program for information about the curves, or display multiple plots in a window environment. Using the modified version of INCA, the controls designer can view describing function curves as well as examine the linear/nonlinear interaction in the Nyquist or Nichols format for possible limit cycling. Also, INCA's graphics interface facilitates the use of the classical gain/phase stabilization concept in nonlinear control synthesis.

The development of nonlinear control synthesis methodology, especially for the design of on-off controller for flexible spacecraft, is perhaps the most important contribution of this project. The developed SIDF software was used to demonstrate the practicality of the developed nonlinear control synthesis methodology. This procedure was used to compare various linear compensators of the INTELSAT 5 yaw-axis RCS controller.

Step #1: Rigid Body Compensation - The type of compensation for the rigid body depends on the performance specifications. This compensation selects the system bandwidth, which is related to the rigid body settling time; in general, a lead-lag filter is used where direct-rate feedback is not possible. The pole and zero locations (i.e., the lead/lag time constant ratio) should be one order of magnitude apart so that the filter does not increase the high frequency gain too much.

Step #2: Pulse Modulator Synthesis - If this item is not fixed as a result of spacecraft design, thought must be given to the original configuration of the modulator. Certain factors which must be considered in the design such as ease of hardware implementation, stabilization requirements of flexible modes, limitations of the reaction control system (i.e. - thruster combinations, thrust level). Another consideration is how closely can the nonlinear controller be approximated as a linear element. For example, it is difficult to tell by looking at the parameters of the PWPF modulator, what its gain and phase characteristics are. Without this knowledge, it is difficult for the designer to correlate linear analysis with nonlinear simulation if the modulator is assumed to have gain of one and zero-degree phase change. This information can be arrived at through describing function analysis of the nonlinearity.

Step #3: Linear Analysis - Linear Analysis methods such as root locus and frequency response will enable the control designer to investigate the stability of the higher-order flexible modes. Higher-order compensation may be necessary, in addition to the rigid-body controller, to compensate for unstable modes.

Step #4: Describing Function Analysis - Information about the nonlinearity, as approximated in a linear sense, can be obtained through the describing function method. By viewing the inverse of the describing function, $-1/N(X,\omega)$, and the linear plant, $G(j\omega)$, in the frequency domain, information can be obtained about nonlinear stability. For example, if the curves intersect, then a high probability for a limit cycle exists; whether or not one occurs depends on the type of input into the system as well as other factors. If there is no intersection between the curves, approximate phase and gain margins can be defined depending on the relative locations of the two arcs. It is important to remember that for energy-storage type nonlinearities, the frequency of the intersection point must be the same on both curves.

Step #5: Nonlinear Simulation - The results in step #4 can be verified by a nonlinear simulation, a capability not yet implemented within INCA but which can be accomplished through an independent external program. The appearance of a limit cycle can be verified in this manner; so can the approximate nonlinear stability margins. Delaying the signal a fixed number of cycles throughout the simulation is a simple way of introducing phase lag into the system. The phase lag, ϕ, follows the relation $\phi = 57.3\ \omega T$, where ω (rad/sec) is the frequency of the point of the linear plant close to the describing function and T (sec) is the time delay. By varying the time delay, the designer can often induce a limit cycle; the smallest value of ϕ which causes a limit cycle can be interpreted as the **nonlinear phase margin.** A similar procedure can be followed for the **nonlinear gain margin.**

Step #6: Linear Analysis Iteration - If the designer is not satisfied with nonlinear stability margins or the possibility of limit cycling, the process can be repeated from step #3 by altering or adding to the linear compensation. If possible, the controller itself can be modified to further decrease the probability of a limit cycle.

The nonlinear simulation presented in this paper was performed external to INCA. A major recommendation of this study is the inclusion of a nonlinear simulation capability within INCA. In that way, describing function and nonlinear simulation analyses, both integral parts of the nonlinear control synthesis methodology, can be combined in the same software.

A detailed report, which covers these topics in specific detail, is currently under preparation and will be available late this year.

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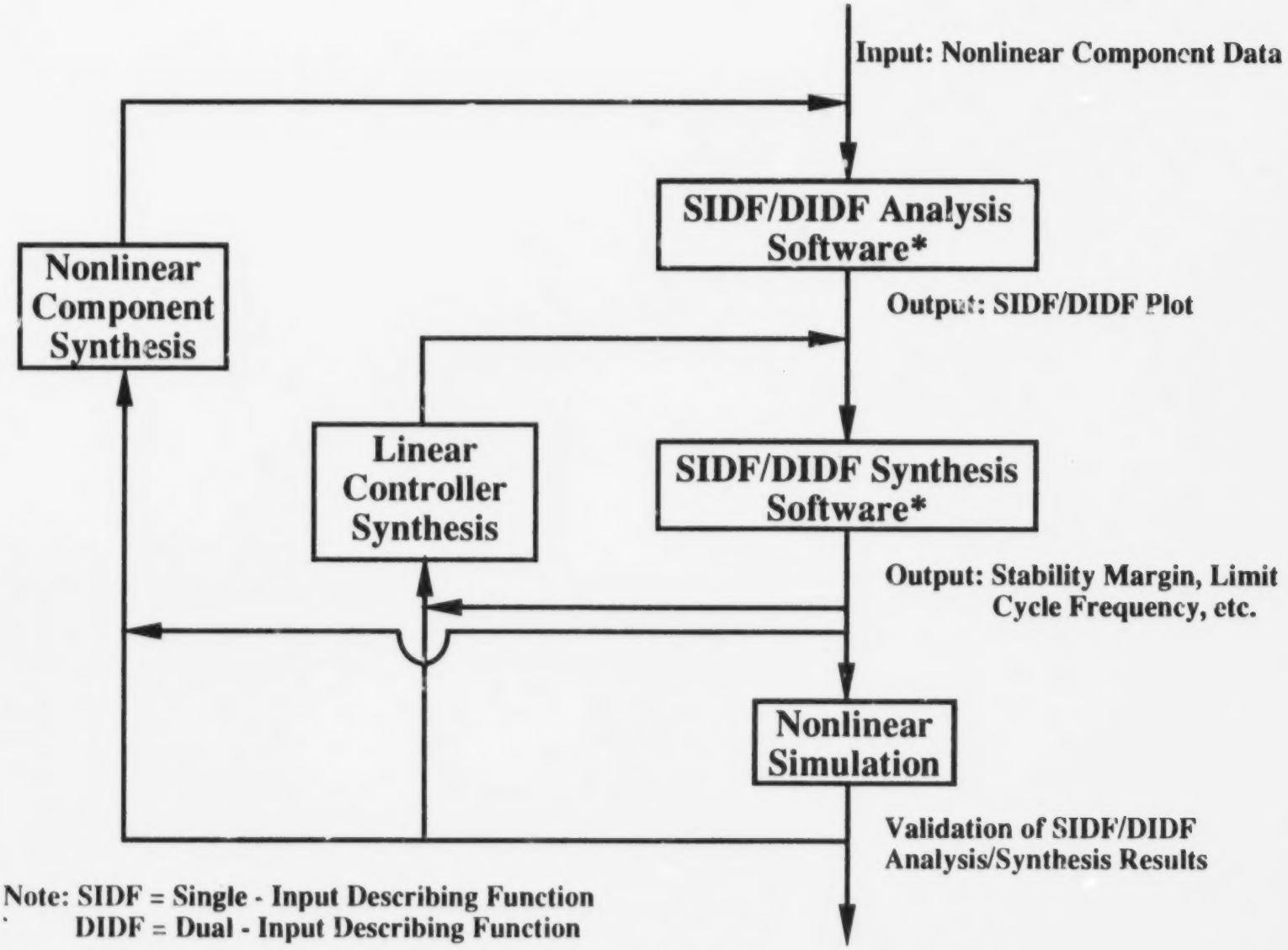
The diagram on the facing page illustrates the methodology developed in this study for the design and analysis of nonlinear control systems. As can be seen in this figure, the ultimate goal of this project is to develop an integrated approach/software for the nonlinear control systems design.

The user obtains single-input describing function (SIDF) or dual-input describing function (DIDF) plots for a particular nonlinear controller, using **SIDF/DIDF Analysis Software**. By combining this data with the plant transfer function and linear controller, using **SIDF/DIDF Synthesis Software**, a control designer can obtain information about the system's tendency towards limit cycling. Predictions can be made about the limit cycle frequency and amplitude or about the phase and/or gain changes which would possibly cause limit cycling. The results can be verified by **Nonlinear Simulation** using numerical simulation software. If a limit cycle is nominally present, the amplitude and frequency can be compared to the predicted values; if a limit cycle is not present, stability margins can be verified. By performing **Linear Compensator Synthesis** using the limit cycle results, a control designer can adjust the compensation accordingly to ensure adequate gain and phase margins to prevent nonlinear instability. The numerical simulation data may lead the designer to conclude that no changes in the linear compensation will prevent limit cycling; in that case, **Nonlinear Component Synthesis** would be required to reconfigure the nonlinear controller to reshape the describing function thereby decreasing the probability of limit cycling.

Since any techniques for nonlinear control systems will almost surely involve approximations, computer simulation to check the analytical and/or graphical designs is an appropriate final step for nonlinear control design. Hence, nonlinear simulation software package integrated to the nonlinear control analysis/synthesis software needs to be developed in the future.

Nonlinear Control Analysis/Synthesis/Simulation



Note: SIDF = Single - Input Describing Function
DIDF = Dual - Input Describing Function

* to be developed in this project

In view of the fact that the synthesis of nonlinear control systems is still in the state of development, no attempt is made in this project to develop new techniques for the analysis and synthesis of general nonlinear systems. Instead, existing techniques such as the describing function method and linear frequency-response methods are employed for the development of a computer-aided approach to nonlinear control systems design.

The specific tasks to be performed in this project basically consists of two parts: (1) Development of computer-aided SIDF analysis/synthesis methodology (Tasks 1 &2) and (2) Development of computer-aided DIDF analysis and synthesis methodology. Products from the study will include a computer software package which will aid the control designer in generating SIDFs and DIDFs for nonlinear stability analysis and control design.

Task Descriptions

- **Task 1: Development and implementation of computer-aided SIDF analysis methodology**
- **Task 2: Development and implementation of computer-aided nonlinear control synthesis methodology using SIDF**
- **Task 3: Same as Task 1 except the use of DIDF**
- **Task 4: Same as Task 2 except the use of DIDF**

Project Schedule

Task	First Year 1	2	3	4	Second Year 1	2	3	4
1. Develop SIDF Analysis Methodology/Software	———							
2. Develop SIDF Synthesis Methodology/Software	———	———	———	*				
3. Develop DIDF Analysis Methodology/Software					———	———	———	
4. Develop DIDF Synthesis Methodology/Software					———	———	———	#

* an interim report plus the developed software package

final report plus the developed software package

Tasks 1 & 2 Accomplishments

- Implemented SIDF analysis algorithms for various nonlinear control elements, including deadzone, limiter, quantizer, Schmitt trigger, and pulse modulators, by utilizing the INCA.

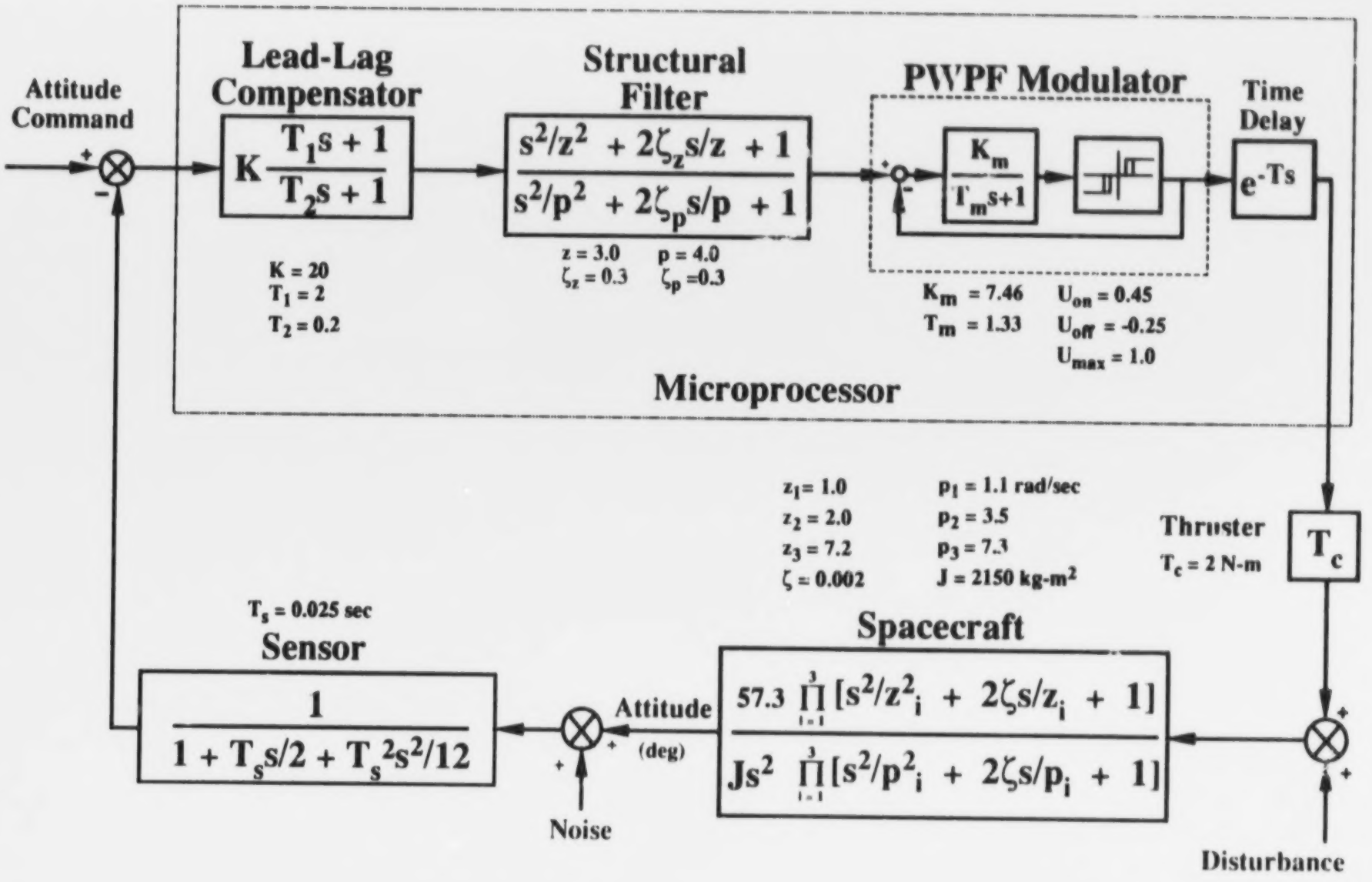
- Enhanced INCA's linear analysis capability by implementing the describing function methodology for nonlinear control analysis.

- Developed a nonlinear control analysis/synthesis methodology for the design of an on-off control system in the presence of interactions between structural modes and nonlinear controller.

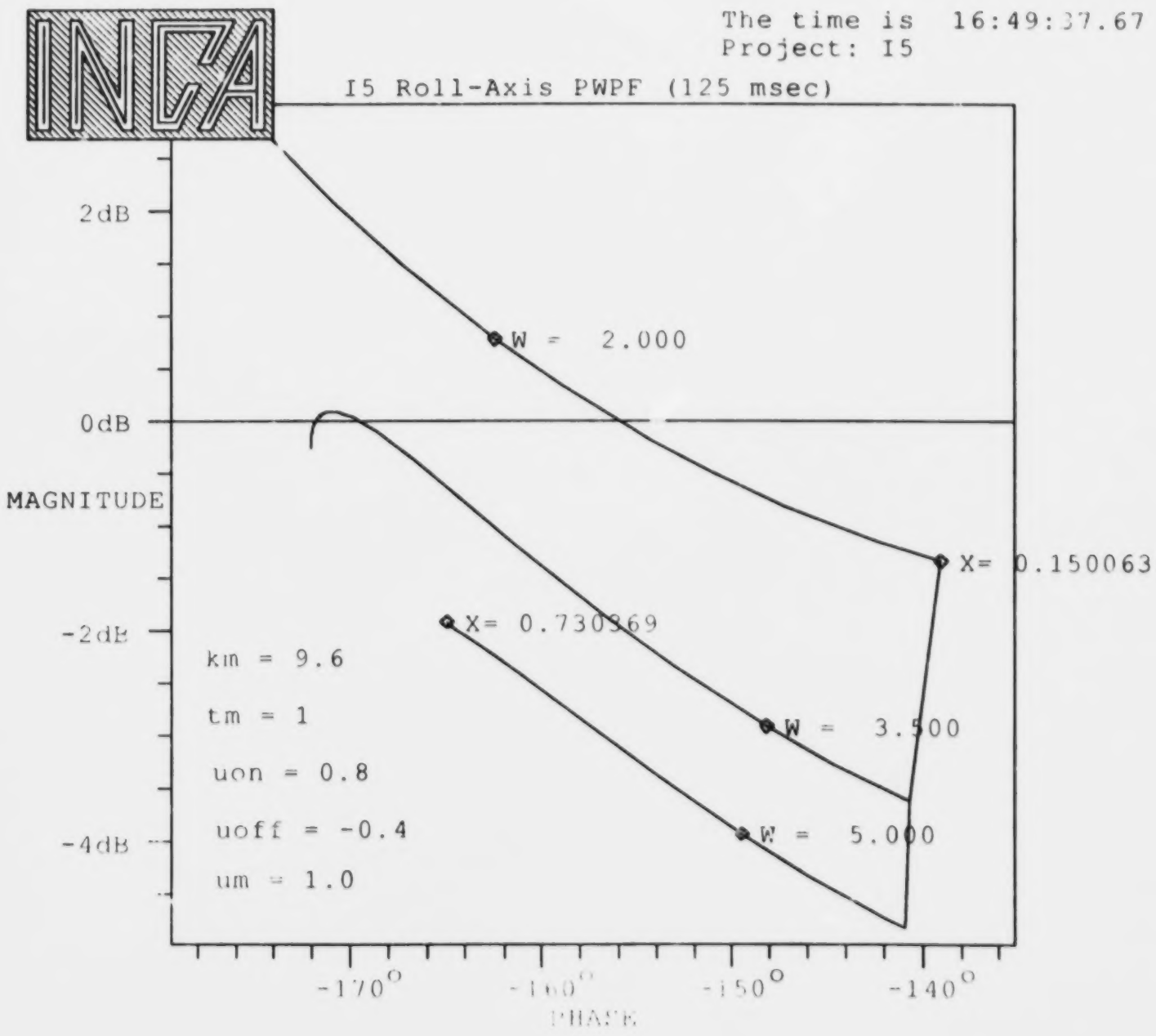
- Demonstrated the practicality of the developed software and methodology for the design and analysis of an attitude control system of a flexible spacecraft.

This figure represents a generic block diagram for an attitude control system of a spacecraft with structural modes. The values given in the viewgraph are for the INTELSAT 5 spacecraft with analog control system (i.e., no computational delay , T = 0). However, most spacecraft with digital control system can be represented in this format. For example, a similar digital control system has been implemented for the INSAT and ARABSAT spacecraft (see, e.g., **"Attitude Stabilization of Flexible Spacecraft During Stationkeeping Maneuvers," B. Wie and C.T. Plescia, AIAA Journal of Guidance, Control, and Dynamics, Vol.7, No.4, July-August 1984).**

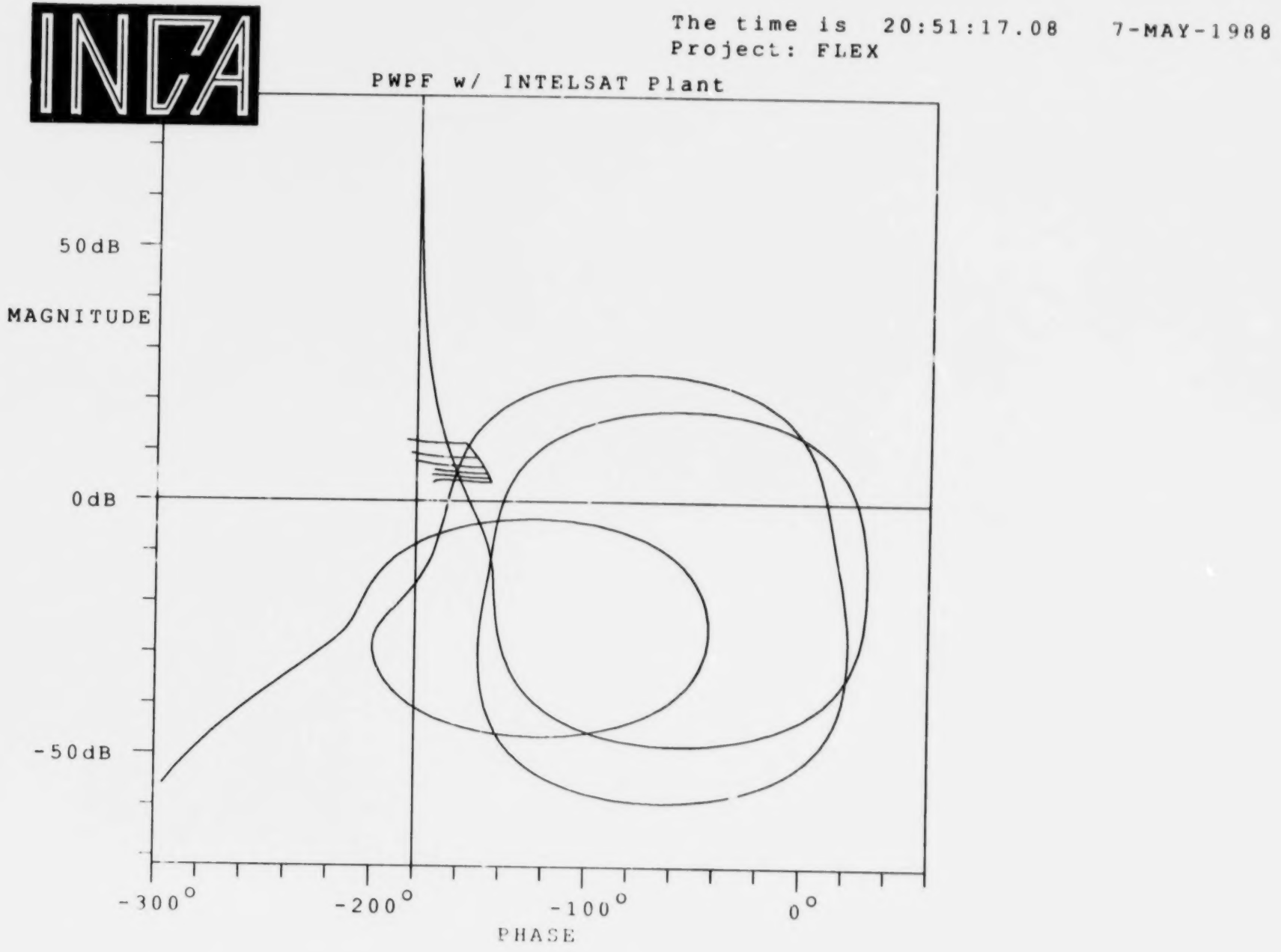
Pulse-Modulated On-Off Control System for a Flexible Spacecraft


Attitude Command
Lead-Lag Compensator
$K\frac{T_1 s + 1}{T_2 s + 1}$
K = 20
T_1 = 2
T_2 = 0.2
Structural Filter
$\frac{s^2/z^2 + 2\zeta_z s/z + 1}{s^2/p^2 + 2\zeta_p s/p + 1}$
z = 3.0 p = 4.0
ζ_z = 0.3 ζ_p = 0.3
PWPF Modulator
$\frac{K_m}{T_m s+1}$
K_m = 7.46 U_{on} = 0.45
T_m = 1.33 U_{off} = -0.25
U_{max} = 1.0
Time Delay
e^{-Ts}
Microprocessor
z_1 = 1.0 p_1 = 1.1 rad/sec
z_2 = 2.0 p_2 = 3.5
z_3 = 7.2 p_3 = 7.3
ζ = 0.002 J = 2150 kg-m^2
Thruster
T_c = 2 N-m
T_c
T_s = 0.025 sec
Sensor
$\frac{1}{1 + T_s s/2 + T_s^2 s^2/12}$
Attitude (deg)
Noise
Spacecraft
$\frac{57.3 \prod_{i=1}^{3} [s^2/z_i^2 + 2\zeta s/z_i + 1]}{Js^2 \prod_{i=1}^{3} [s^2/p_i^2 + 2\zeta s/p_i + 1]}$
Disturbance

On the opposite page, a describing function plot of the INTELSAT 5 PWPF modulator is shown. This plot is an example of the INCA's describing function capability as a result of modifications made to version 3.0 of the original INCA. INCA allows the user to examine several types of nonlinear elements but the software is versatile enough to enable interactive analysis of each type of describing function. For example, the opposing figure depicts a describing function which is of "memory" type, meaning that it is a function of amplitude as well as frequency of the input. The user is able to query the program for information on the makeup of the curve; information about the frequency and amplitude of the describing function can be obtained during an interactive session. This type of information is essential when analyzing limit cycle interaction between flexible modes and the nonlinear control system.


The time is 16:49:37.67 6-JUL-1988
Project: I5
I5 Roll-Axis PWPF (125 msec)
2dB
0dB
-2dB
-4dB
MAGNITUDE
W = 2.000
X= 0.150063
X= 0.730369
W = 3.500
W = 5.000
km = 9.6
tm = 1
uon = 0.8
uoff = -0.4
um = 1.0
-170°
-160°
-150°
-140°
PHASE

The opposing plot depicts an example of nonlinear controller interaction with spacecraft's structural modes. The figure shows two sets of curves: a PWPF modulator describing function and a four-mode linear plant (i.e., rigid body mode plus three flexible modes). It is seen that the two sets of curves intersect at two points. The horizontal lines of the describing function represent different input frequencies to the nonlinearity; in order for a limit cycle to exist, the frequency of the points of intersection on both curves must be equivalent. At this point, it is not possible to tell what range of frequencies the describing function represents and whether or not a limit cycle is present. However, similar plots are discussed in greater detail in this report.


INCA
The time is 20:51:17.08 7-MAY-1988
Project: FLEX
PWPF w/ INTELSAT Plant
50dB
MAGNITUDE
0dB
-50dB
-300°
-200°
-100°
0°
PHASE

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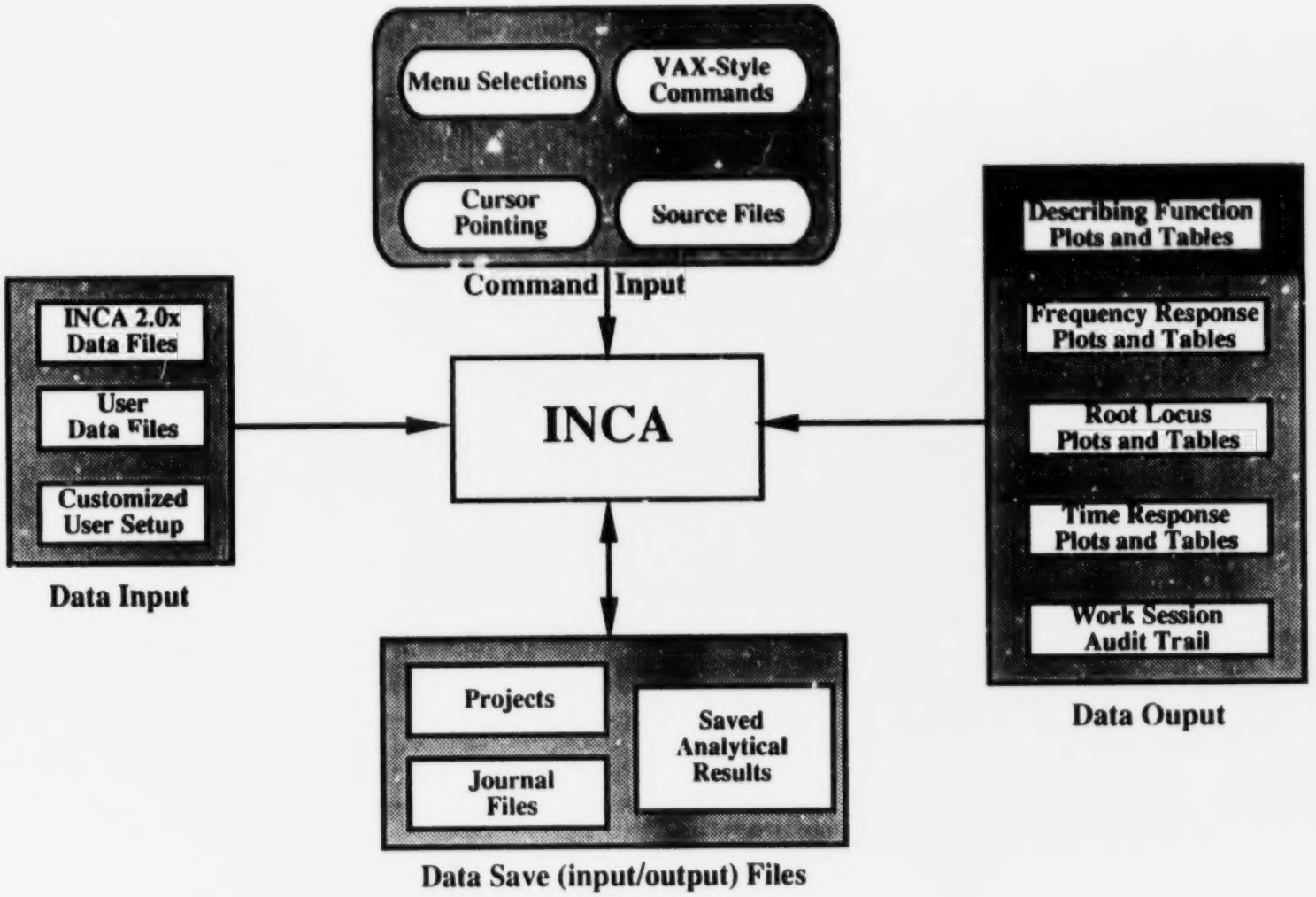
INCA is the result of a software development effort out of the Guidance and Control Branch (Code 712) at NASA Goddard Space Flight Center. The chief developer of the software is Dr. John Downing while the effort is being directed by Frank Bauer. The latest version of INCA, known as version 3.0, is formally distributed through COSMIC meaning that it is non-commercial software. INCA allows the designer to perform linear controls analysis in a menu-driven, interactive graphics environment; the user can define transfer functions, in either **s**, **z**, or **w** domain, in symbolic form where root locus, frequency response, or time response analyses may be performed. Furthermore, with the use of a high-resolution graphics terminal, a user can query the program for information about the curves, or display multiple plots in a window environment.

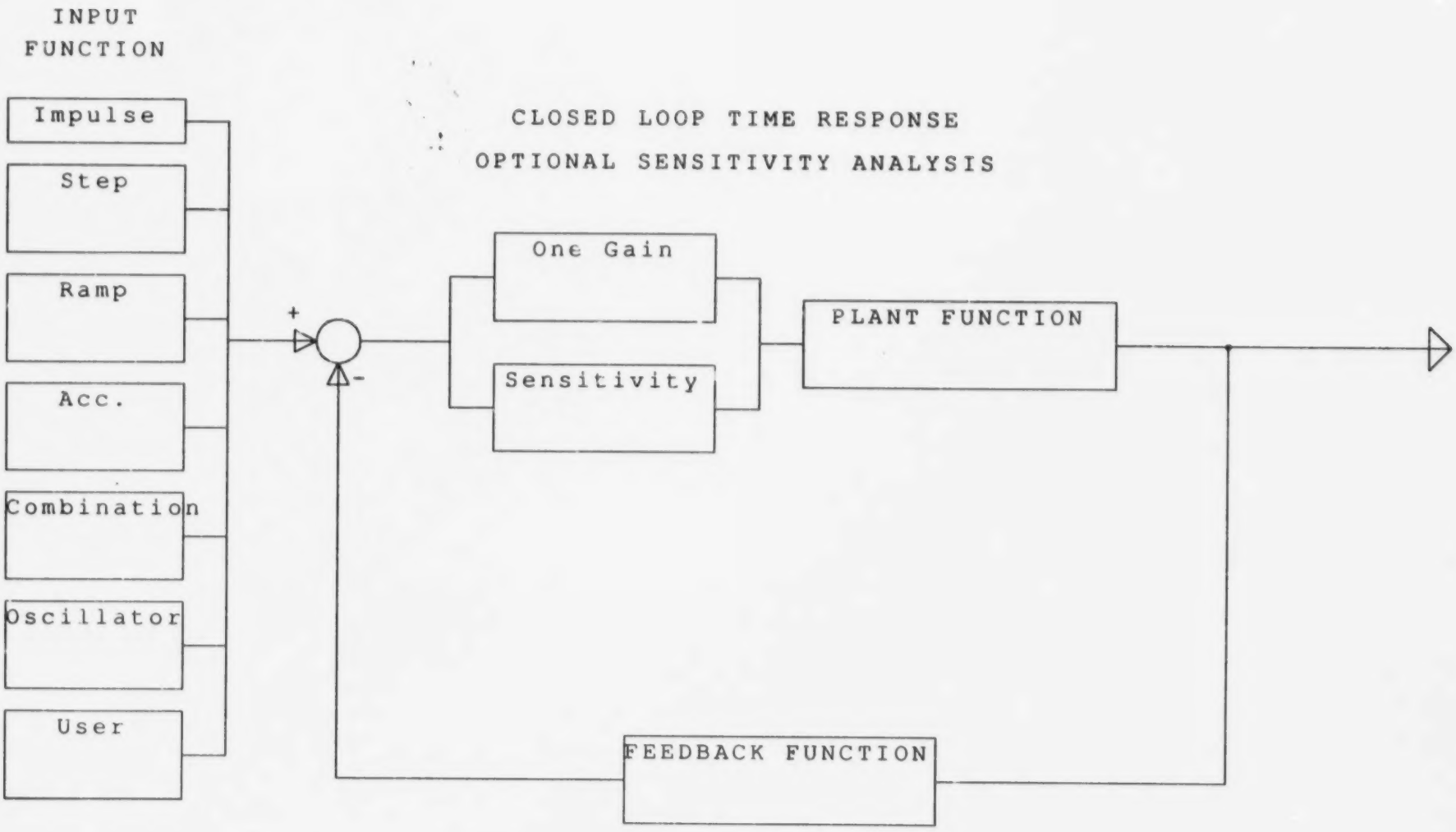
The adaptations accomplished in this project involve inclusion of the describing function capability into version 3.0 of INCA. Members of the project are grateful for the technical assistance given by Dr. Downing and Mr. Bauer who offered help and ideas for the framework of the describing function modifications.

INCA Data & Command Flow*



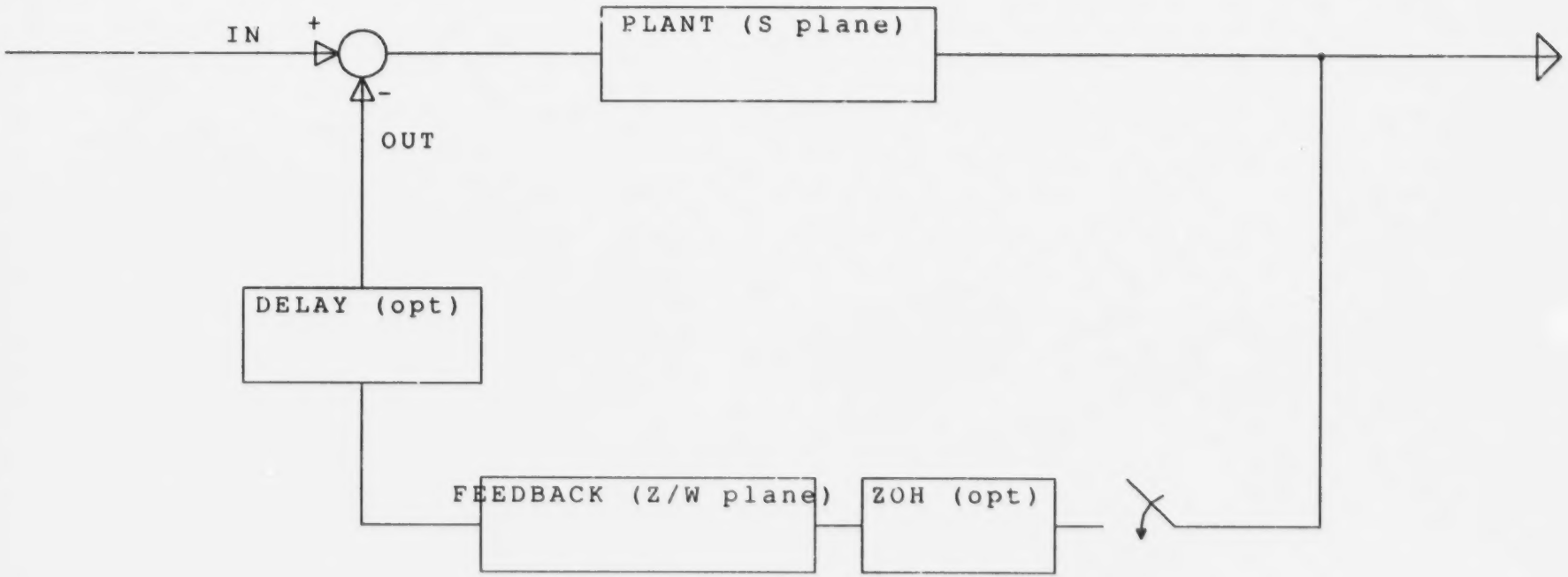
*Adapted from diagram found in INCA 3.0 Primer

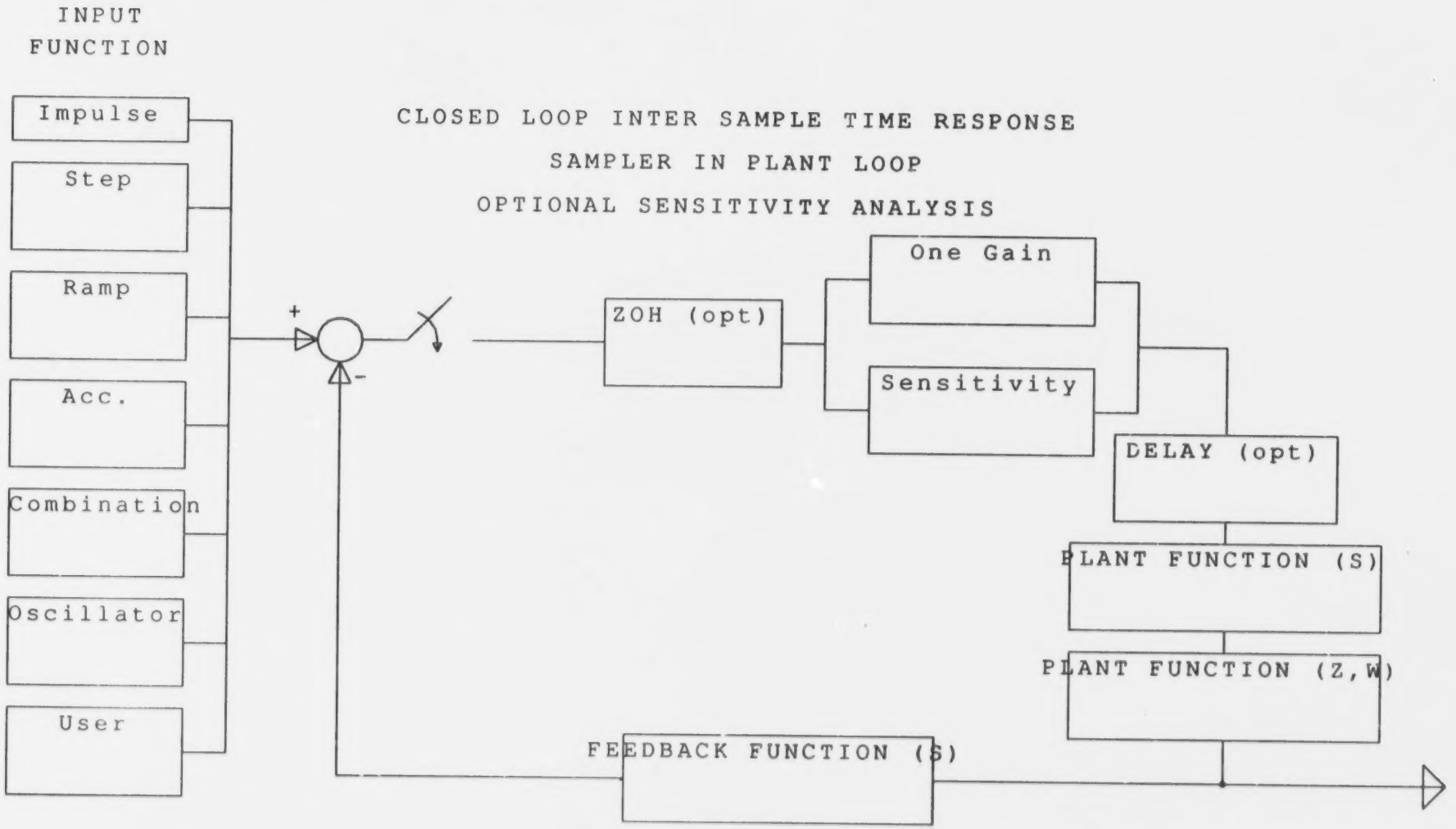
The following four plots represent the graphic interface the INCA user has with the program. These plots are taken from the time response and frequency response analyses for linear control systems. The program queries the user for the inputs to the systems shown; the values then appear in the block diagram form shown here. This interface allows the user to obtain an overall perspective of linear control analysis and design.

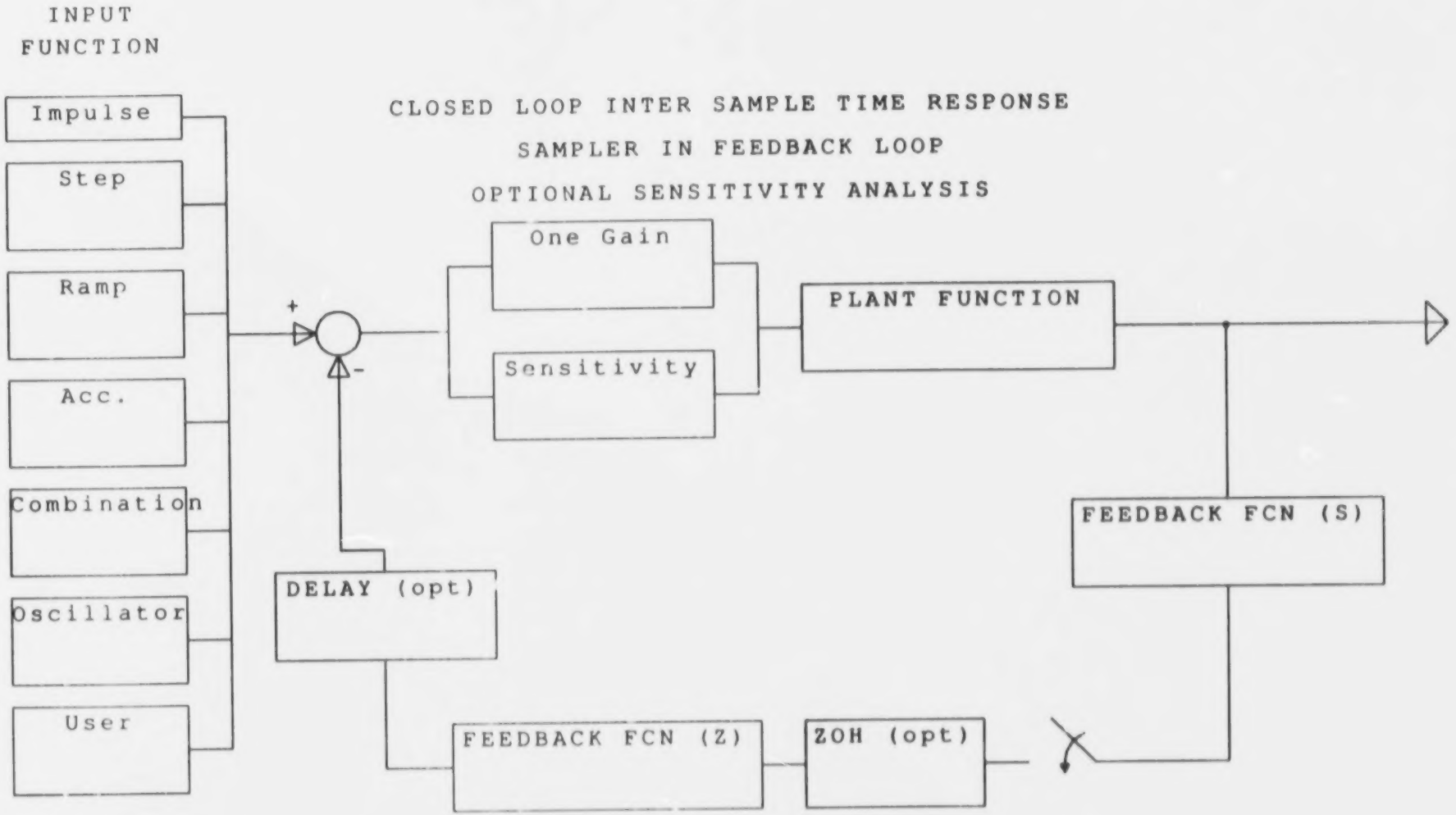

INPUT
FUNCTION
Impulse
Step
Ramp
Acc.
Combination
Oscillator
User
CLOSED LOOP TIME RESPONSE
OPTIONAL SENSITIVITY ANALYSIS
+
-
One Gain
Sensitivity
PLANT FUNCTION
FEEDBACK FUNCTION

FREQUENCY RESPONSE STAR OPERATOR

OPEN LOOP TRANSFER FUNCTION = OUT/IN




INPUT
FUNCTION
Impulse
Step
Ramp
Acc.
Combination
Oscillator
User
CLOSED LOOP INTER SAMPLE TIME RESPONSE
SAMPLER IN PLANT LOOP
OPTIONAL SENSITIVITY ANALYSIS
+
-
ZOH (opt)
One Gain
Sensitivity
DELAY (opt)
PLANT FUNCTION (S)
PLANT FUNCTION (Z,W)
FEEDBACK FUNCTION (S)


INPUT
FUNCTION
CLOSED LOOP INTER SAMPLE TIME RESPONSE
SAMPLER IN FEEDBACK LOOP
OPTIONAL SENSITIVITY ANALYSIS
Impulse
Step
Ramp
Acc.
Combination
Oscillator
User
+
-
One Gain
Sensitivity
PLANT FUNCTION
FEEDBACK FCN (S)
DELAY (opt)
FEEDBACK FCN (Z)
ZOH (opt)

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INCA supports describing function analysis of several types of nonlinearities. Many of these are found in attitude control systems; implementation of other common elements is also being planned. The nonlinearities found on the facing page are known as "memory-less" systems where the describing functions are not a function of frequency, just amplitude.

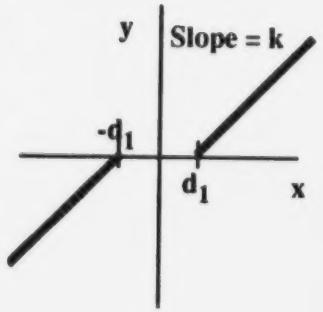
Deadzone - The deadzone represents a system where there is no output for input signals between d_1 and d_2. This nonlinearity is often found in systems such as the case where two loosely-meshed gears experience a small angle of rotation from the nominal position where no torque is applied to either gear. This type of nonlinearity is often found in payload pointing systems.

Limiter - The limiter is an extremely common nonlinearity found in most electronic and mechanical systems. In this element, the amplitude of an input signal is modified in a proportional manner when the input signal amplitude is between d1 and -d1; at that point, the output of the system is constant as shown in the opposing figure. In attitude control systems, for example, the output of torque of a momentum/reaction wheel is limited by the capabilities of the reaction motor; the device can produce a torque linearly proportional to the input signal until a certain point after which the wheel acceleration, and therefore applied torque, is limited.

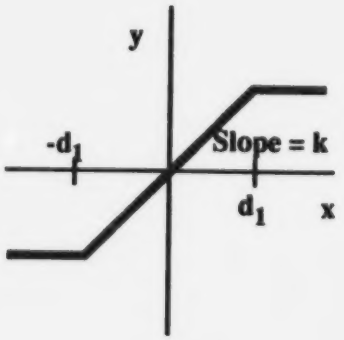
Relay with Deadband- The relay controller has three output states; zero, and a constant value whose sign is determined by the sign of the input signal. As seen on the opposite, the output signal is zero, if the input signal is between -d1 and d1, and equal to +A or -A depending on the sign of x. Also, the relay scheme is an approximation of many attitude thruster control laws where attitude error angle is the input and thruster torque is the output; as a result, the relay is also known as the "bang-off-bang" control law. The **quantizer** is a form of the relay where there are more than three levels of output.

Schmitt Trigger - The Schmitt trigger is often found in spacecraft attitude control systems using reaction jets. the Schmitt trigger logic can be represented as a relay with proportional positive feedback.

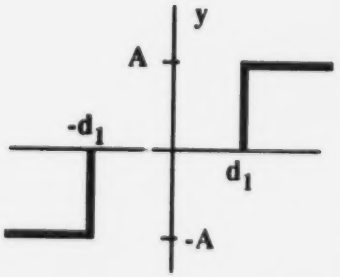
Nonlinear Control Elements



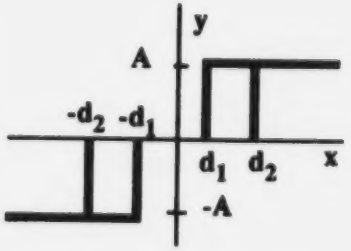
Deadzone



Limiter



Relay with Deadband



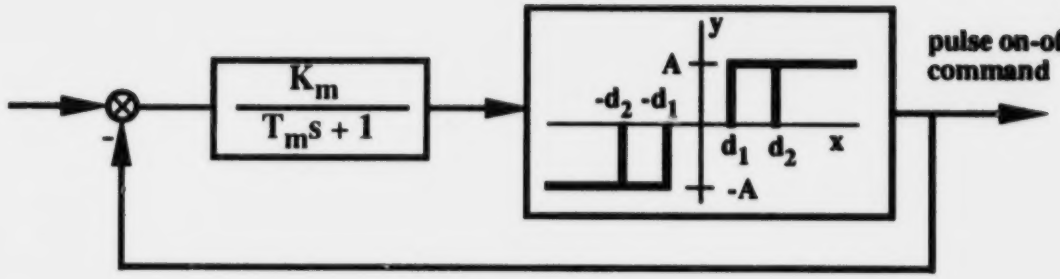
Schmitt Trigger

The type of nonlinearities presented on the next two pages are also found in spacecraft attitude control systems. These nonlinear elements are known as "memory" type since they produce describing functions which are functions of frequency as well as amplitude.

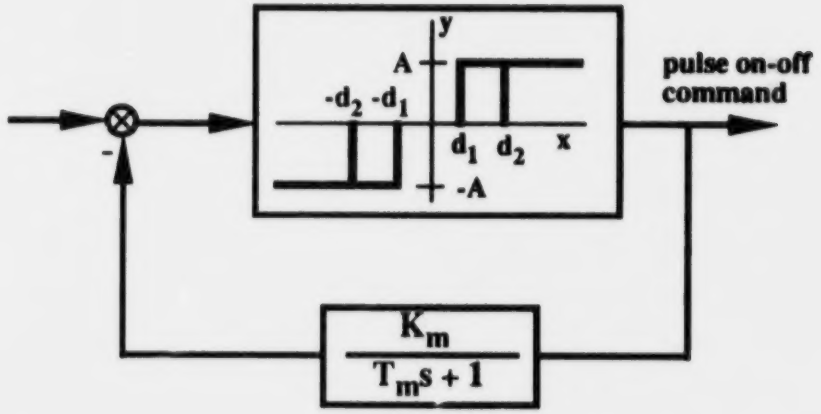
PWPF Modulator - A generic version of the PWPF (pulse-width and pulse-frequency) modulator is shown on the facing page. This device has been successfully used for reaction jet control systems of many communications satellites, including the INTELSAT 5, INSAT, and ARABSAT spacecraft. As indicated in the figure, the PWPF Modulator consists of a Schmitt trigger connected in series with a first-order filter and negative feedback loop. For a constant input, the PWPF modulator output represents an on-off pulse sequence with a linear duty cycle proportional to the input.

Derived-Rate Modulator - As depicted in the figure, the DRM is similar to the PWPF modulator in appearance. Furthermore, the DRM produces a pulse signal whose duty cycle is linearly related to the input amplitude in the same manner as the PWPF modulator. The static characteristics of the DRM are similar to those of the PWPF modulator. Despite their similarities, the two devices have different gain and phase characteristics; the DRM introduces phase lead into a system while the PWPF Modulator introduces phase lag.

Nonlinear Control Elements (cont'd)



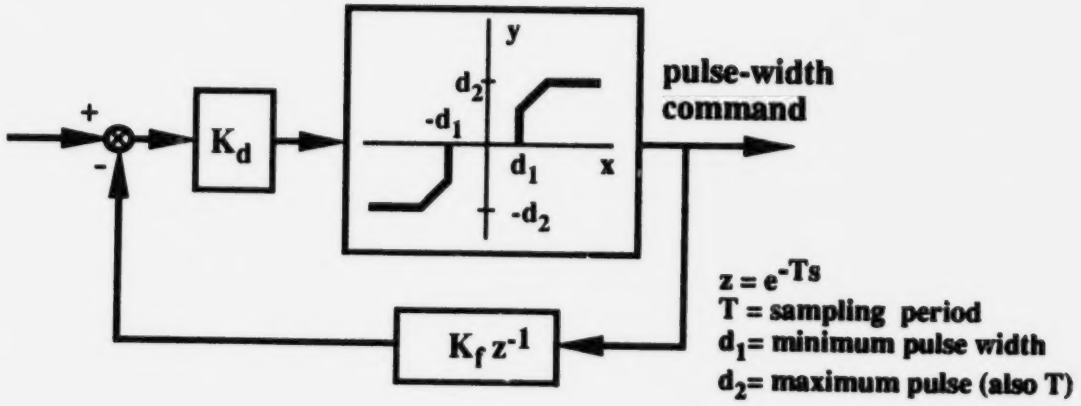
PWPF Modulator



Derived-Rate Modulator

Pulse-Width Modulator - The PWM differs from the previous devices in that it is primarily employed for digital controller. Even so, the device's static characteristics can be analyzed similarly to the PWPF modulator and the DRM. This device is being considered for the Orbital Maneuvering Vehicle (OMV) Reaction Control System. Like the modulators previously discussed, the PWM introduces phase lead into the control loop; the amount of phase lead is usually small and is dependent on the relative sizes of the input frequency and the feedback gain . If the feedback signal inside the PWM loop is relatively large compared to the input to the modulator, the delayed signal will introduce damping when the PWM input amplitude is within the vicinity of the relay.

Nonlinear Control Elements (cont'd)



Pulse-Width Modulator

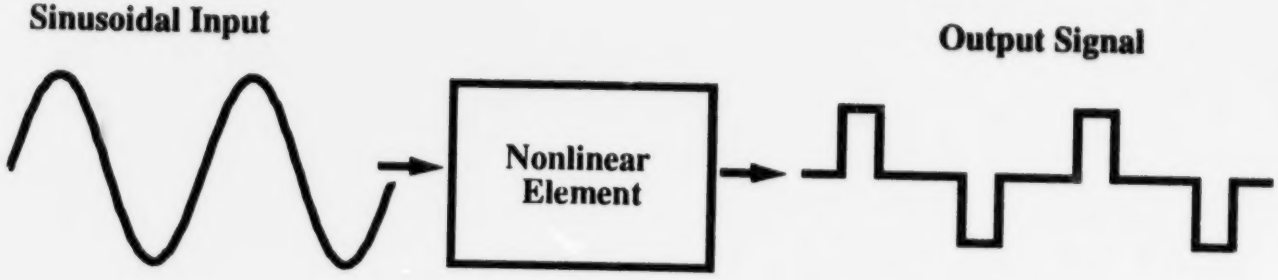
No attempt is made here to discuss the describing function method in rigor or depth, since this topic is treated extensively in many texts (e.g., see **Multi-Input Describing Functions and Nonlinear System Design**, Gelb and Vander Velde, McGraw-Hill, 1968). We will instead emphasize the basic principles and fundamental assumptions, and discuss a computer-aided approach to nonlinear control design using the describing function method.

The basic idea of the describing function method for analyzing nonlinear control systems is to replace the nonlinear element with an approximate linear model whose "gain" is a function of the input amplitude. While the describing function method is inherently inexact and is formulated in the frequency domain, there is a growing recognition that the describing function method has capabilities which are not simply available in any other approach. The describing function method has also recently received great attention since the control researchers are coming to recognize that the frequency-response method is a very powerful approach for dealing with robustness, unmodelled dynamics, and other considerations (see, e.g., "A Systematic Nonlinear Controller Design Approach based on Quasilinear System Models," Taylor, J.H., Proceedings of 1983 American Control Conference, June 1983).

In the describing function analysis, we assume that only the fundamental harmonic component of the output is significant. Such an assumption is often valid since the higher harmonics in the output of a nonlinear element are often smaller amplitude than the amplitude of the fundamental harmonic component. If no energy-storage element is included in the nonlinear devices, then the SIDF is a function only of the input amplitude. On the other hand, if energy-storage element is included (e.g., pulse modulators), then the SIDF is a function of both the amplitude and frequency of the input.

Describing Function Method

- Describing function method involves analyzing gain and phase changes between the sinusoidal input to nonlinearity and the output signal.



- The SIDF (Single-Input Describing Function) is defined to be the complex ratio of the fundamental harmonic component of the output to the sinusoidal input.

- Limit cycle amplitude, X, and frequency, ω, can be determined by intersection of describing function, $N(X, j\omega)$, and linear plant transfer function, $G(j\omega)$, in frequency domain;

$$N(X, j\omega)G(j\omega) = -1 \quad \text{or} \quad -1/N(X, j\omega) = G(j\omega)$$

Describing Function Method (Cont'd)

Advantages

- Allows linear representation of nonlinear elements.
- Permits determination of limit cycling without extensive numerical simulations.
- Enables user to determine nonlinear stability margins.

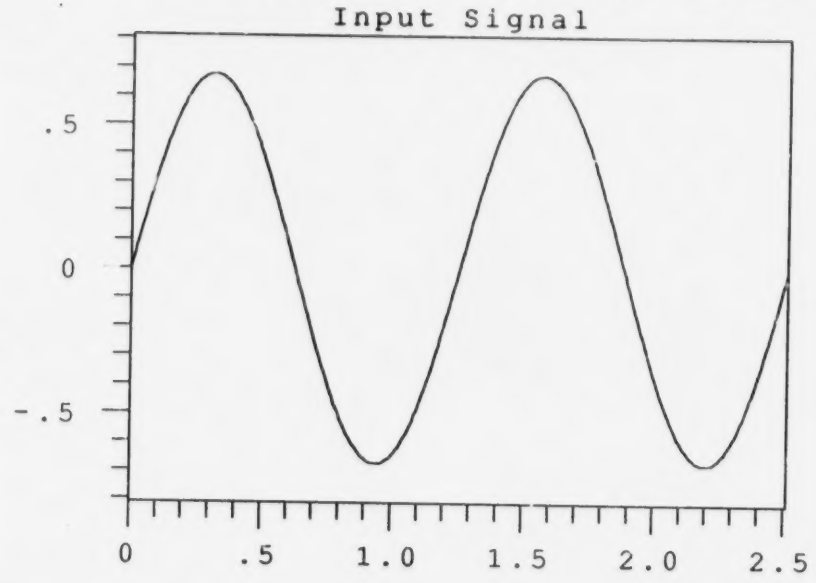
Disadvantages

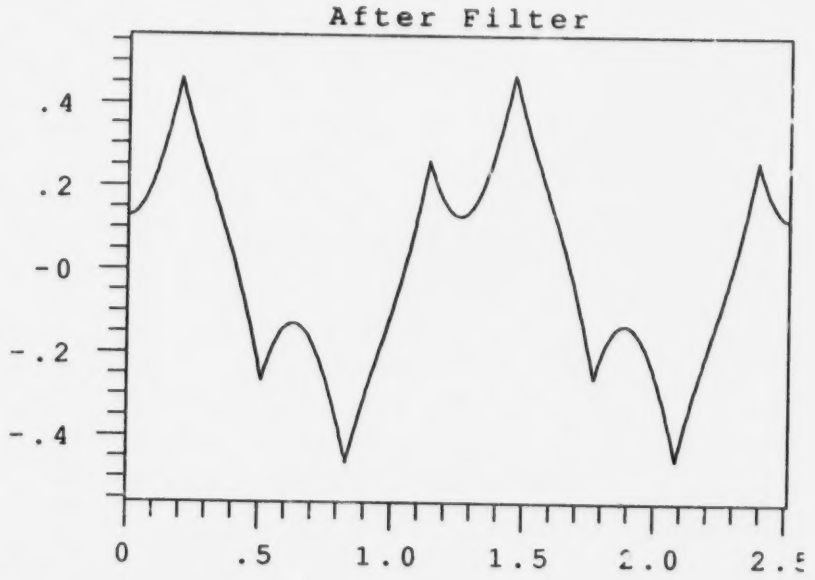
- Sinusoidal assumption of input signal is not always accurate and the method is inherently inexact.
- First harmonic approximation of output sinusoid is not always accurate.
- Limit cycle prediction accuracy depends on orthogonality of intersection of describing function and linear transfer function curves.

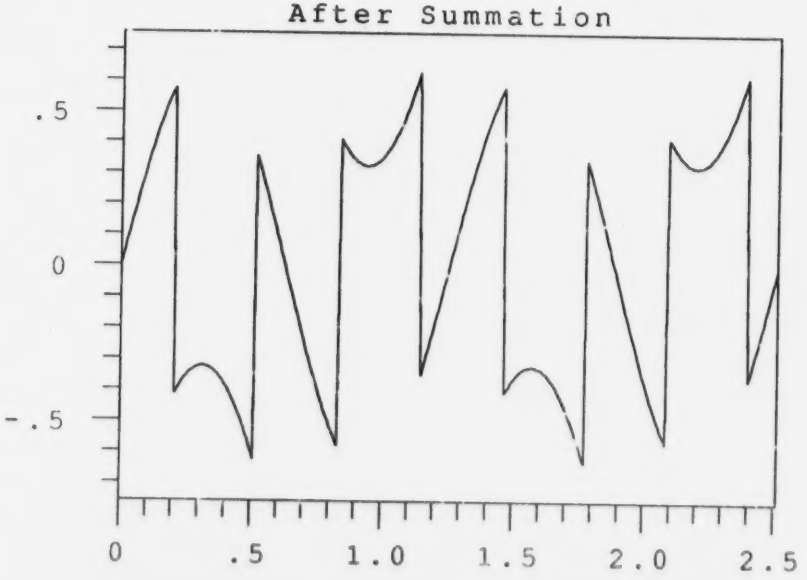
The four plots on the facing page show four different states of the PWPF modulator. Shown in the upper left is the input signal which corresponds to a sinusoid of $\omega = 5$ rad/sec and amplitude, X, equal to .676. This corresponds to the leftmost point on the $\omega = 5$ curve in the describing function on the previous page. The plot on the lower left represents the input to the lag filter; the plot on the upper right depicts the output of that filter which is fed into the Schmitt trigger. The Schmitt trigger output, and by definition the modulator output, is square-wave type curve in the graph on the lower right. Superimposed on that curve is the fundamental Fourier harmonic which is fitted to the actual pulse output.

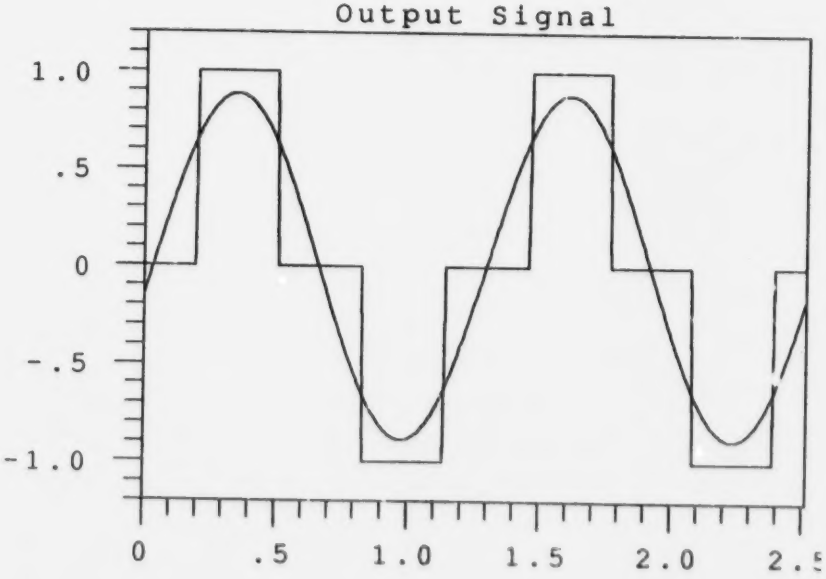
File: 676.DAT The time is 11:33:48.17 16-JUL-1988

PWPF Mod. States for X=0.676









Here, the input signal and the first harmonic representation of the output signal is shown. The difference in amplitude, gain change, and the change in phase are shown here as well. The gain and phase values define the complex value of the describing function N for $\omega = 5$ and $X = 0.676$. The gain and phase values for $-1/N$ are plotted on the previous Nichols chart. INCA performs this calculation for the range of user-defined amplitudes and frequencies.

Once an SIDF representation of a nonlinear element is obtained, it may be used in several ways. The most traditional SIDF analysis is seeking limit cycle conditions. Using the describing function $N(X, j\omega)$, the limit cycle frequency and amplitude are obtained as the solution to the equation $N(X, j\omega)G(j\omega) = -1$ where $G(j\omega)$ is the transfer function of the linear part of of the system. In many cases, solving this equation analytically or graphically is extremely tedious and complicated. Nonlinear control synthesis to improve the stability margins or to modify the limit cycle frequency are also very time consuming. In this project, we have implemented the graphical approach in the describing function method to the INCA program.

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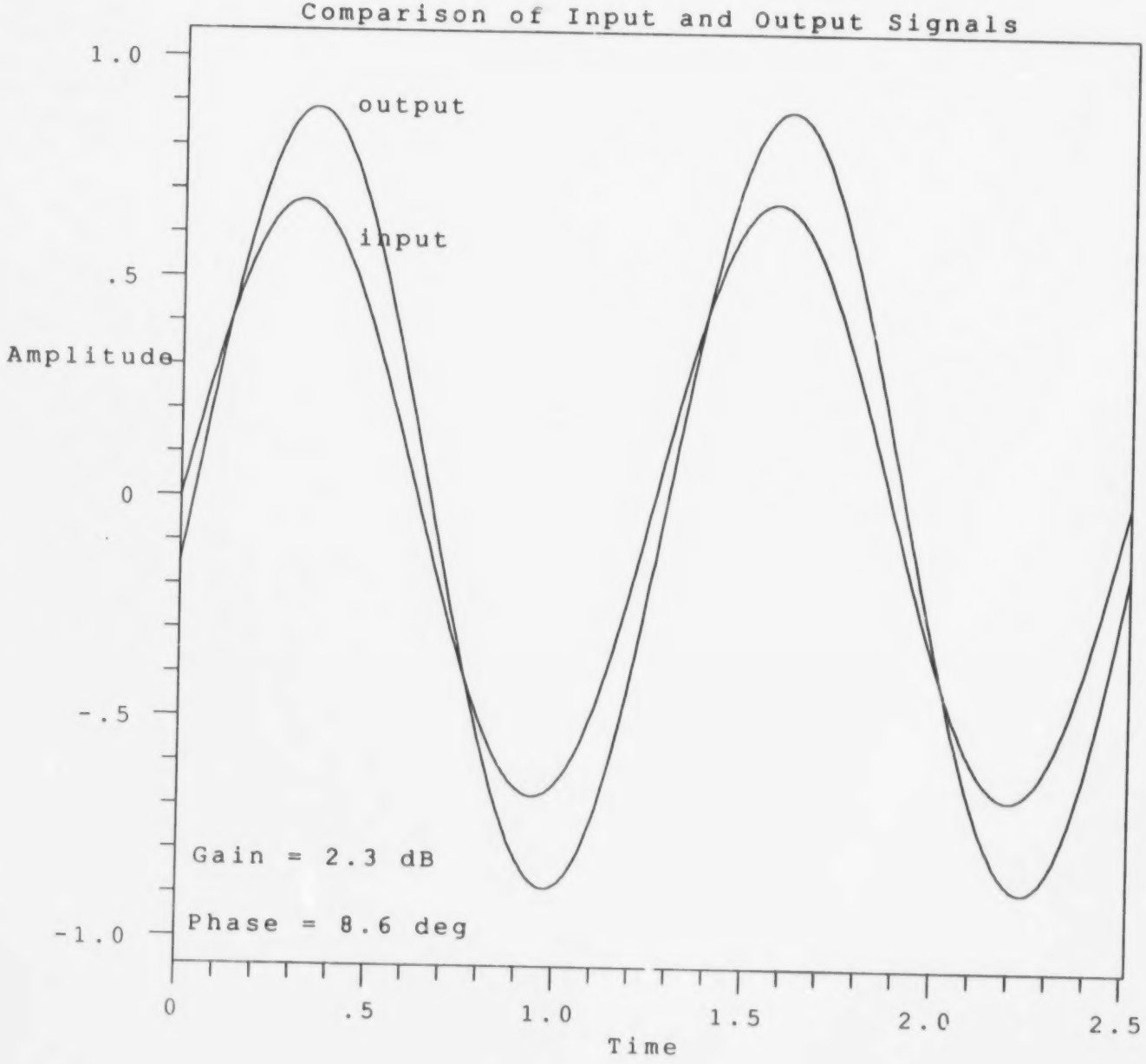


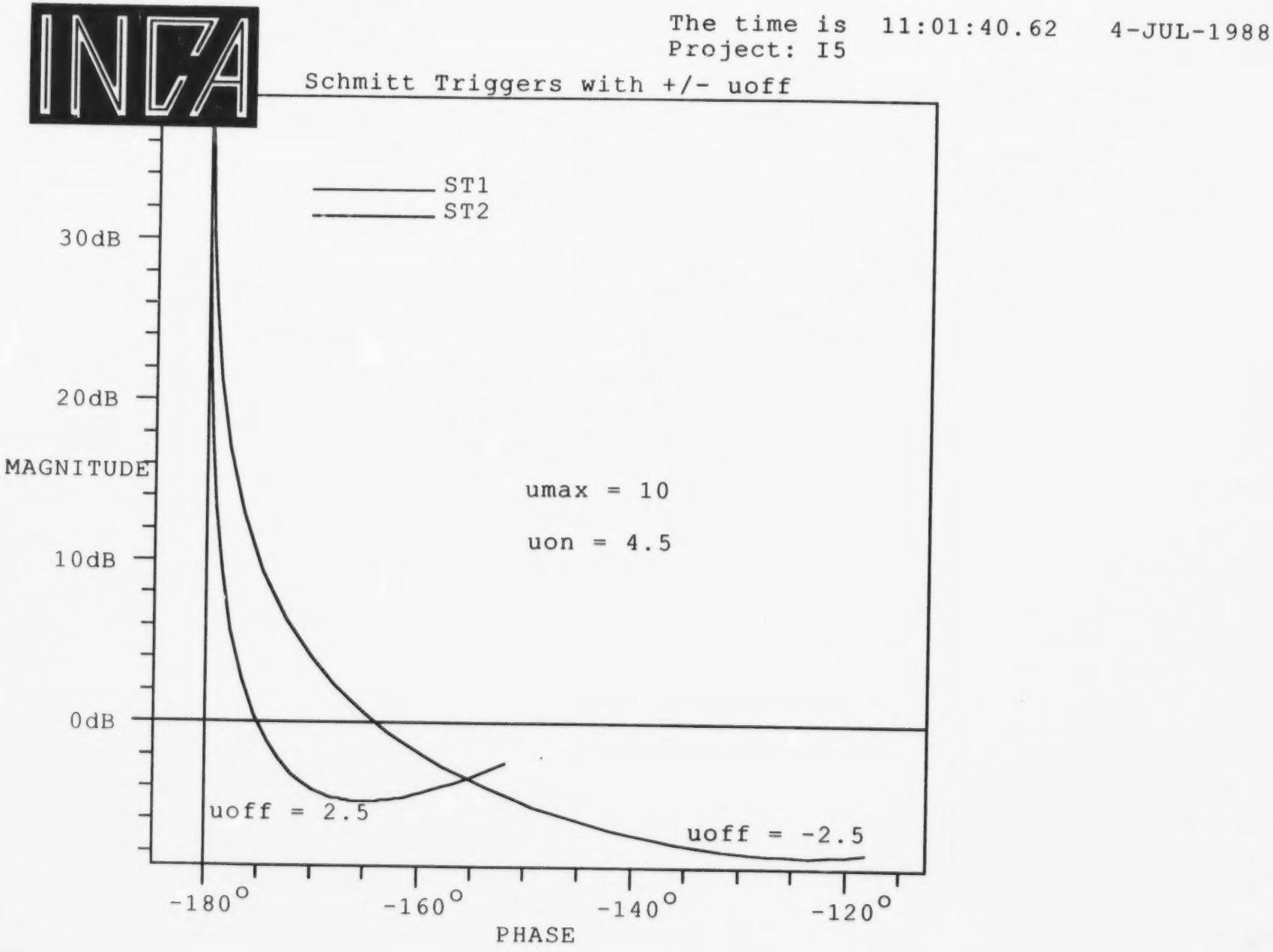
INCA Enhancement for SIDF Analysis

- INteractive Controls Analysis (INCA) program version 3.0 released January '88 from NASA/GSFC:
  - Symbolic manipulation of transfer functions
  - Interactive graphics interface:
    - o Root Locus (includes sensitivity analysis capability)
    - o Frequency Response (Nyquist, Nichols, Bode)
    - o Time Response of linear systems

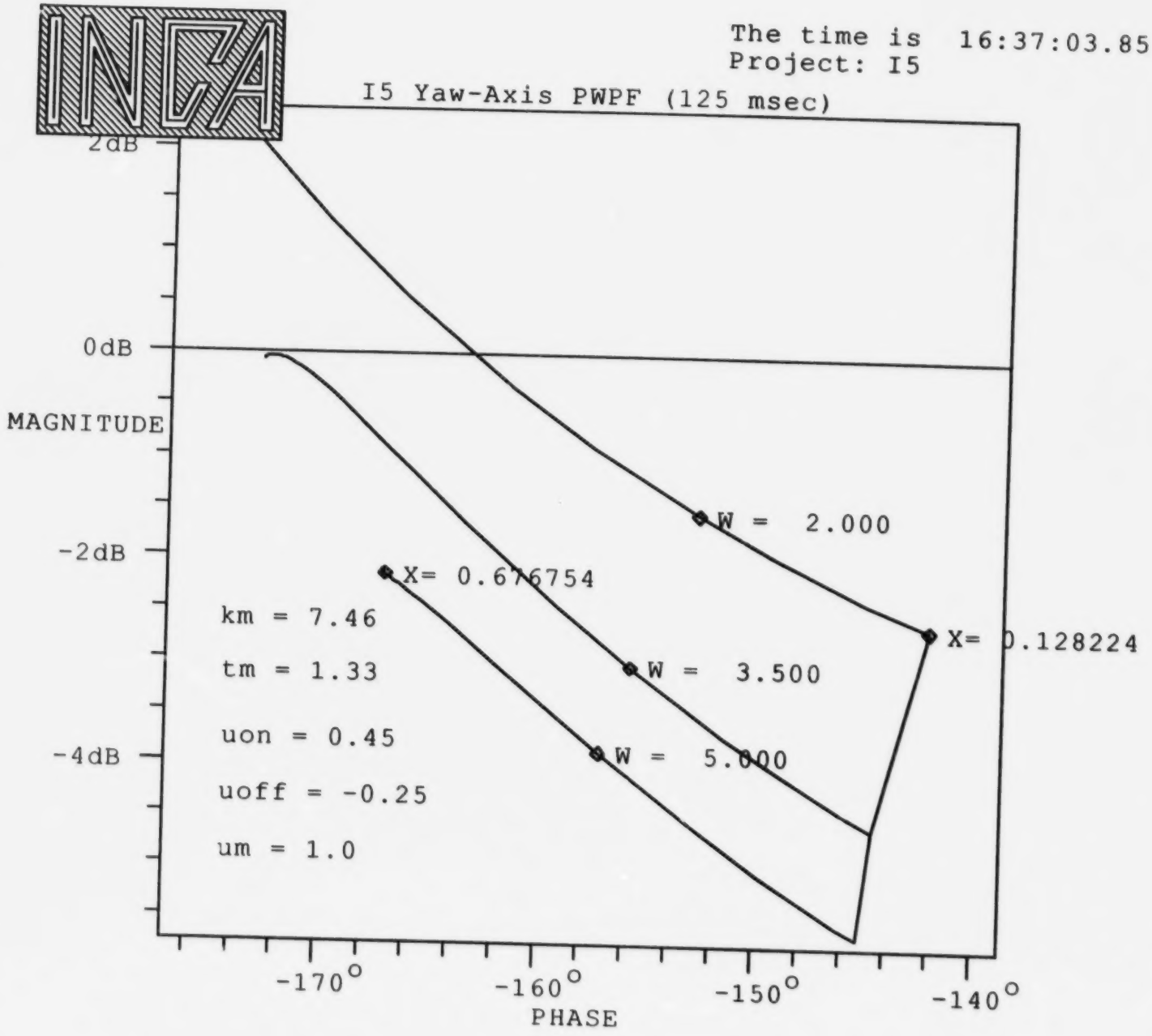
- INCA has been enhanced to calculate and graphically display describing functions:
  - Memoryless systems: deadzone, limiter, quantizer, ST
  - Memory systems (energy storage): PWPF modulator

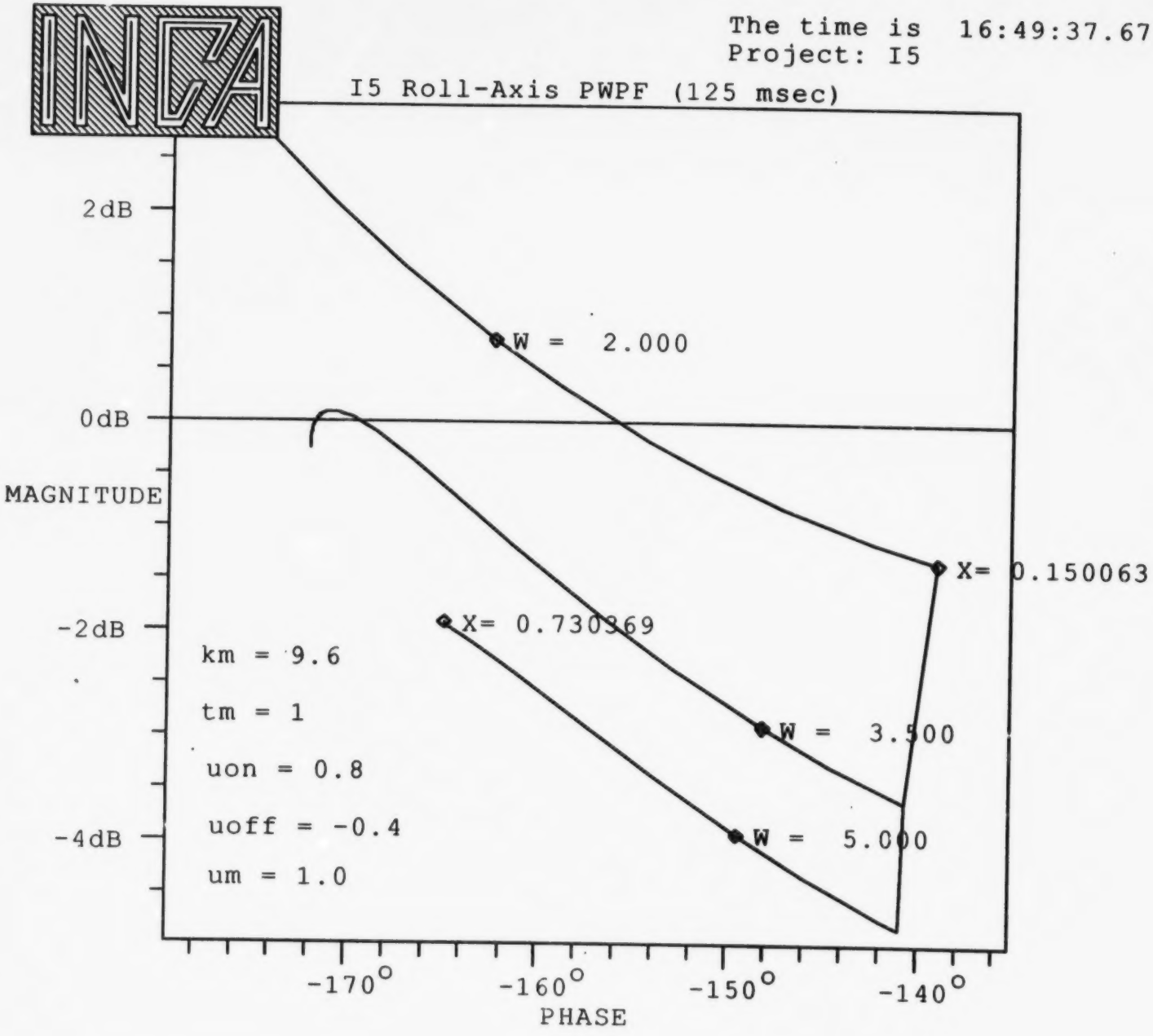
The Schmitt trigger and the PWPF modulator are the only nonlinearities represented in INCA that impart phase shift to the input sinusoid; as a result, they are easier to view on the INCA screen for they lie off the -180° line. For the deadzone, quantizer, and the limiter it is difficult to observe on a black-and-white plot for they lie directly on the -180° line.

The plot on the opposite page compares two Schmitt trigger describing functions. The Schmitt trigger curve has the characteristic hook shape with the smallest amplitude at the extreme end of the curve; it is observed from the plot that at low amplitudes, the trigger imparts greater phase lead. Also, the two curves represent different types of hystereses. Both triggers have the same output level (u_{max}) and pulse-on level (u_{on}), but the pulse-off levels (u_{off}) are inverse of on another. An interesting point to note is the fact that the Schmitt trigger with negative u_{off} imparts more phase lag at lower values of input amplitude than the trigger with positive u_{on}. It can be surmised that the wider separation between u_{on} and u_{off} yields a greater phase shift for input sinusoids of low amplitude.

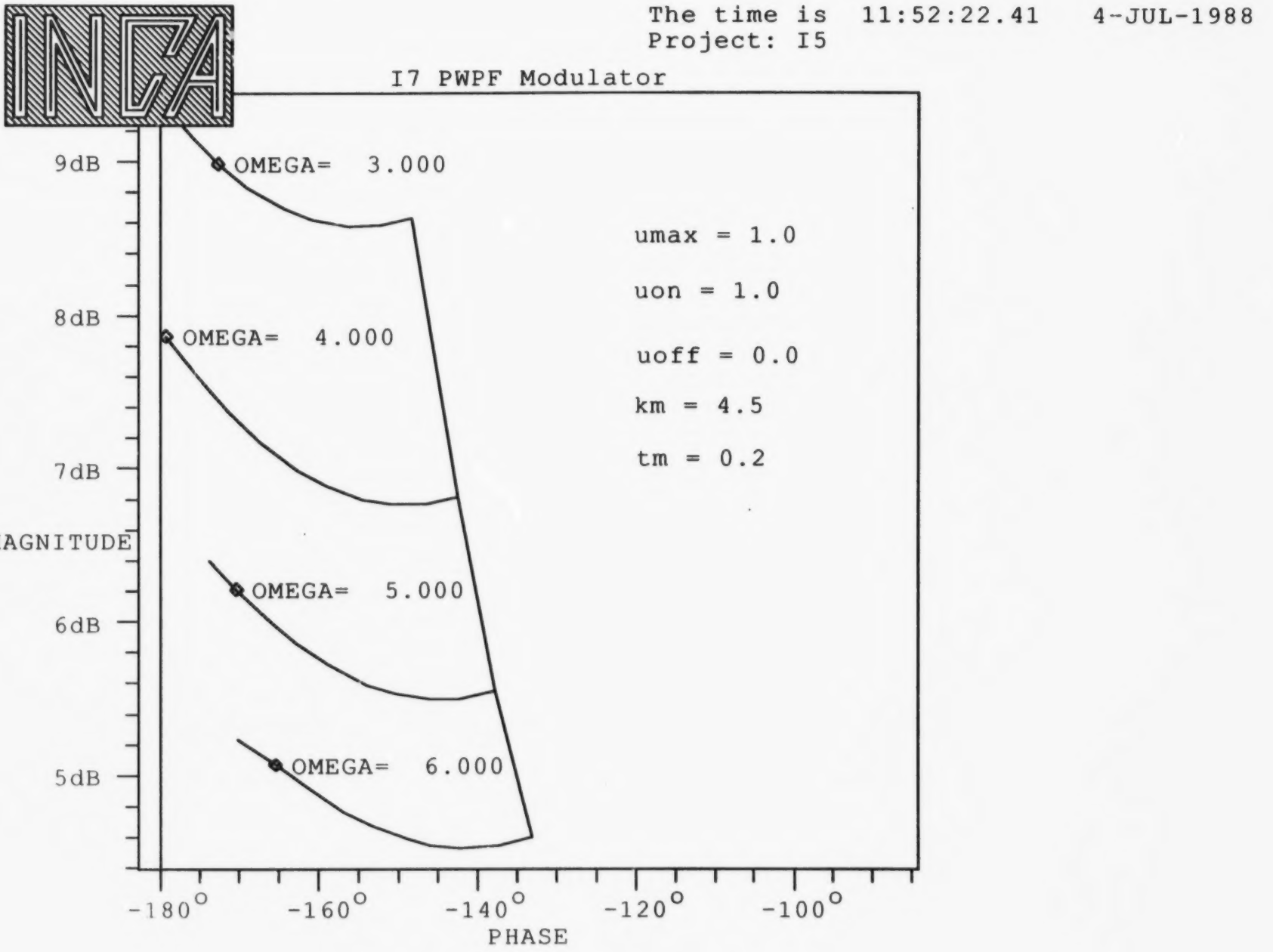

INCA
The time is 11:01:40.62 4-JUL-1988
Project: I5
Schmitt Triggers with +/- uoff
ST1
ST2
30dB
20dB
MAGNITUDE
10dB
0dB
umax = 10
uon = 4.5
uoff = 2.5
uoff = -2.5
-180°
-160°
-140°
-120°
PHASE

The describing function depicted here is the yaw-axis PWPF modulator for the INTELSAT 5 (I5) spacecraft. The plot can be separated into two parts. The roughly vertical line at the right of the plot is known as the minimum-amplitude line. This line represents an absolute boundary of the describing function. The series of curves slanting up to the left of the graph denote the parts of the describing function determined by altering the input frequency into the nonlinearity. As explained previously, the PWPF modulator describing function is dependent on input amplitude as well as input sinusoidal frequency. On the graph, the frequency range of the describing function is $\omega = 2$, 3.5, and 5 rad/sec. The curves terminate at a value of amplitude where unlike the Schmitt trigger, the PWPF modulator describing function exists over an finite area on the Nichols chart. The curves terminate on the left-hand side of the graph; this is actually not the case. It is only at that point that INCA stops plotting the curve for a specific reason discussed on the next few viewgraphs.


The time is 16:37:03.85 6-JUL-1988
Project: I5
I5 Yaw-Axis PWPF (125 msec)
2dB
0dB
-2dB
-4dB
MAGNITUDE
W = 2.000
X= 0.676754
X= 0.128224
W = 3.500
W = 5.000
km = 7.46
tm = 1.33
uon = 0.45
uoff = -0.25
um = 1.0
-170°
-160°
-150°
-140°
PHASE


INCA
The time is 16:49:37.67 6-JUL-1988
Project: I5
I5 Roll-Axis PWPF (125 msec)
2dB
0dB
-2dB
-4dB
MAGNITUDE
W = 2.000
X= 0.150063
X= 0.730369
W = 3.500
W = 5.000
km = 9.6
tm = 1
uon = 0.8
uoff = -0.4
um = 1.0
-170°
-160°
-150°
-140°
PHASE

The time is 11:52:22.41 4-JUL-1988
Project: I5



Contents

- Introduction and project overview
  - Task descriptions
  - Project schedule
  - Tasks 1 & 2 accomplishments
- Introduction to INCA program
- Task 1: SIDF implementation
  - Nonlinear control elements
  - Single-input describing function (SIDF)
  - INCA enhancement for SIDF analysis
- Task 2: Nonlinear control synthesis using SIDF
- Plan for the second year: Tasks 3 & 4
- Summary and conclusions
- Appendix

The development of nonlinear control synthesis methodology, especially for the design of on-off controller for flexible spacecraft, is perhaps the most important contribution of this project. The developed SIDF software was used to demonstrate the practicality of the developed nonlinear control synthesis methodology. This procedure was used to compare various linear compensators of the INTELSAT 5 yaw-axis RCS controller.

Step #1: Rigid Body Compensation - The type of compensation for the rigid body depends on the performance specifications. This compensation selects the system bandwidth, which is related to the rigid body settling time; in general, a lead-lag filter is used where direct-rate feedback is not possible. The pole and zero locations (i.e., the lead/lag time constant ratio) should be one order of magnitude apart so that the filter does not increase the high frequency gain too much.

Step #2: Pulse Modulator Synthesis - If this item is not fixed as a result of spacecraft design, thought must be given to the original configuration of the modulator. Certain factors which must be considered in the design such as ease of hardware implementation, stabilization requirements of flexible modes, limitations of the reaction control system (i.e. - thruster combinations, thrust level). Another consideration is how closely can the nonlinear controller be approximated as a linear element. For example, it is difficult to tell by looking at the parameters of the PWPF modulator, what its gain and phase characteristics are. Without this knowledge, it is difficult for the designer to correlate linear analysis with nonlinear simulation if the modulator is assumed to have gain of one and zero-degree phase change. This information can be arrived at through describing function analysis of the nonlinearity.

Step #3: Linear Analysis - Linear Analysis methods such as root locus and frequency response will enable the control designer to investigate the stability of the higher-order flexible modes. Higher-order compensation may be necessary, in addition to the rigid-body controller, to compensate for unstable modes.

Nonlinear Control Synthesis Methodology for a Flexible Spacecraft

- **Step #1: Rigid Body Mode Compensation**
  - **Rigid body control bandwidth, settling time, etc.**
  - **Lead-lag filter if direct-rate feedback is not possible.**

- **Step #2: Pulse Modulator Synthesis**
  - **Type of modulator is determined by nature of flexible mode interactions with rigid body control logic.**
    - **o PWPF modulator, Schmitt trigger for gain-stabilization**
    - **o Derived-Rate Modulator for phase-stabilization**

- **Step #3: Linear Analysis**
  - **Employ structural filters, if needed, for structural mode compensation**

Step #4: Describing Function Analysis - Information about the nonlinearity, as approximated in a linear sense, can be obtained through the describing function method. By viewing the inverse of the describing function, $-1/N(X,\omega)$, and the linear plant, $G(j\omega)$, in the frequency domain, information can be obtained about nonlinear stability. For example, if the curves intersect, then a high probability for a limit cycle exists; whether or not one occurs depends on the type of input into the system as well as other factors. If there is no intersection between the curves, approximate phase and gain margins can be defined depending on the relative locations of the two arcs. It is important to remember that for energy-storage type nonlinearities, the frequency of the intersection point must be the same on both curves.

Step #5: Nonlinear Simulation - The results in step #4 can be verified by a nonlinear simulation, a capability not yet implemented within INCA but which can be accomplished through an independent external program. The appearance of a limit cycle can be verified in this manner; so can the approximate nonlinear stability margins. Delaying the signal a fixed number of cycles throughout the simulation is a simple way of introducing phase lag into the system. The phase lag, ϕ, follows the relation $\phi = 57.3\ \omega T$, where ω (rad/sec) is the frequency of the point of the linear plant close to the describing function and T (sec) is the time delay. By varying the time delay the designer can often induce a limit cycle; the value of smallest value of ϕ which causes a limit cycle can be construed as the **nonlinear phase margin.** A similar procedure can be followed for the **nonlinear gain margin.**

Step #6: Linear Analysis Iteration - If the designer is not satisfied with nonlinear stability margins or the possibility of limit cycling, the process can repeated from step #3 by altering or adding to the linear compensation. If possible, the controller itself can be modified to further decrease the probability of a limit cycle.

Nonlinear Control Synthesis Methodology for a Flexible Spacecraft (cont'd)

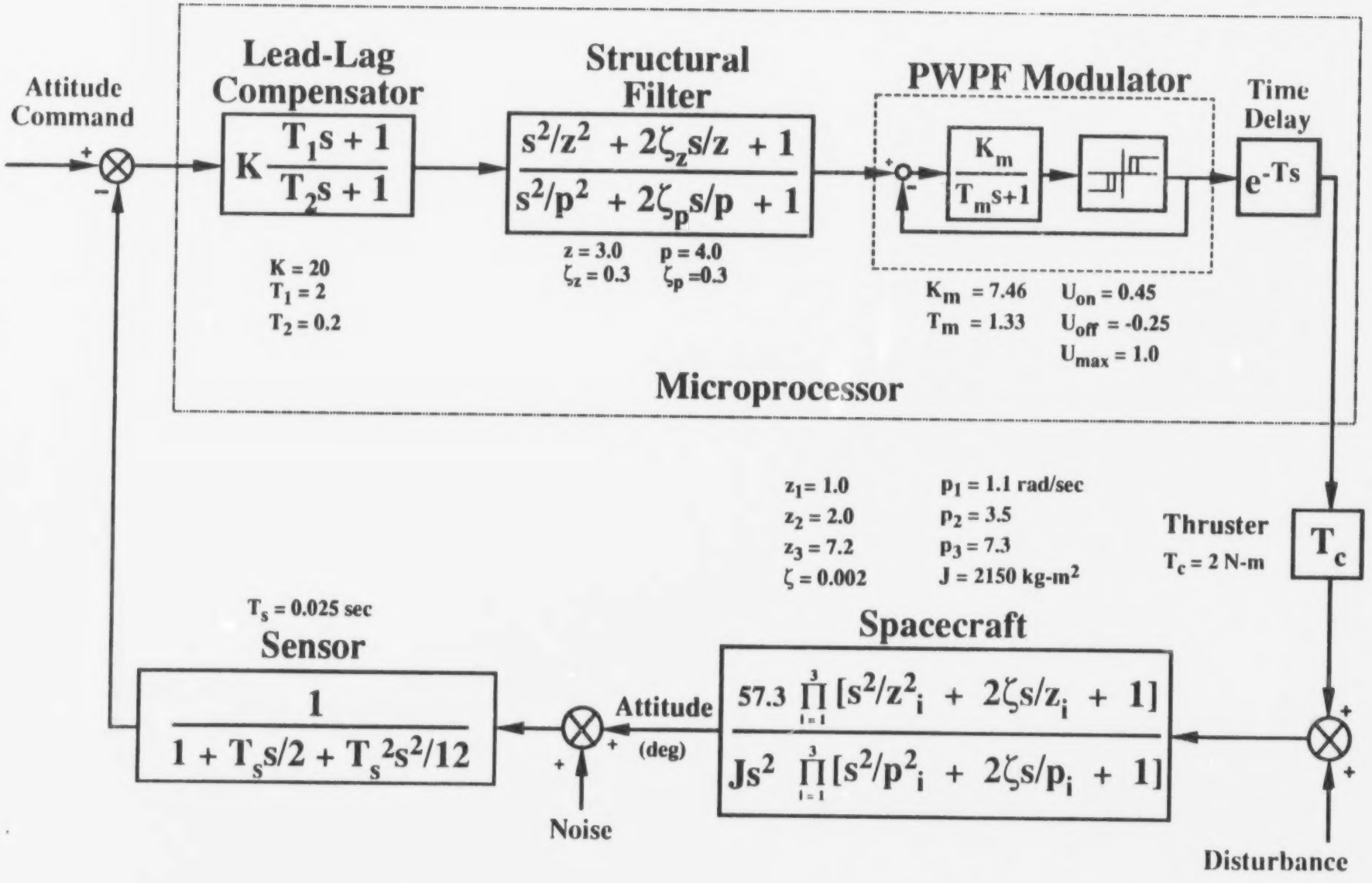
- **Step #4: Describing Function Analysis**
  - If nonlinearity is of memory type, calculate describing function for frequencies corresponding to flexible modes.
  - View describing function curve in Nichols or Nyquist plot.
    - o If linear response and describing function curves intersect, note amplitude and frequency of limit cycle.
    - o If curves do not intersect, observe approximate margins in gain and phase between curves.

- **Step #5: Nonlinear Simulation**
  - If limit cycle was predicted, verify through simulation.
  - If limit cycle was not predicted, verify nonlinear stability margins.

- **Step #6: Linear Analysis Iteration**

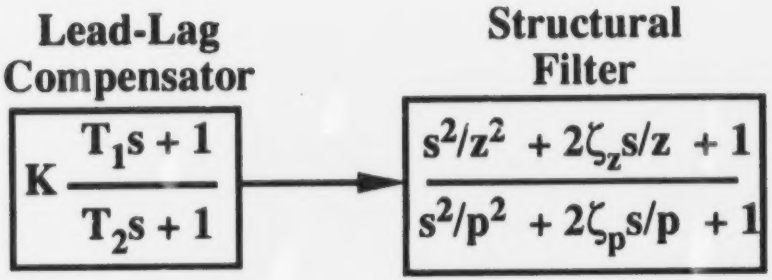
The INTELSAT 5 yaw-axis RCS controller was used to validate the SIDF analysis methodology developed in this project. The information for the plant, linear compensation, and nonlinear controller is listed on the opposing page. The linear compensator parameters listed in the block diagram are for case #1.

Pulse-Modulated On-Off Control System for a Flexible Spacecraft


Attitude Command
Lead-Lag Compensator
$K\frac{T_1 s + 1}{T_2 s + 1}$
K = 20
T_1 = 2
T_2 = 0.2
Structural Filter
$\frac{s^2/z^2 + 2\zeta_z s/z + 1}{s^2/p^2 + 2\zeta_p s/p + 1}$
z = 3.0 p = 4.0
ζ_z = 0.3 ζ_p = 0.3
PWPF Modulator
$\frac{K_m}{T_m s+1}$
K_m = 7.46 U_{on} = 0.45
T_m = 1.33 U_{off} = -0.25
U_{max} = 1.0
Time Delay
e^{-Ts}
Microprocessor
z_1 = 1.0 p_1 = 1.1 rad/sec
z_2 = 2.0 p_2 = 3.5
z_3 = 7.2 p_3 = 7.3
ζ = 0.002 J = 2150 kg-m^2
Thruster
T_c = 2 N-m
T_c
T_s = 0.025 sec
Sensor
$\frac{1}{1 + T_s s/2 + T_s^2 s^2/12}$
Attitude (deg)
Noise
Spacecraft
$\frac{57.3 \prod_{i=1}^{3} [s^2/z_i^2 + 2\zeta s/z_i + 1]}{Js^2 \prod_{i=1}^{3} [s^2/p_i^2 + 2\zeta s/p_i + 1]}$
Disturbance

Five different linear compensators are summarized on the facing page. In cases #1 and 2, simple lead-lag compensation is employed to stabilize both rigid body and flexible modes. Cases #3,4,5 utilized different structural filtering to avoid the structural mode limit cycle predicted by the describing function method.

INTELSAT-5 RCS Design Examples



Case #1: Lead-Lag Compensator without Structural Filtering

- $K=20$, $T_1=2$, $T_2=0.2$

Case #2: Lead-Lag Compensator without Structural Filtering

- $K=20$, $T_1=1$, $T_2=0.1$

Case #3: Lead-Lag Compensator with Phase Lead Filtering

- Case #1 with $z=3$, $\zeta_z=0.3$, $p=4$, $\zeta_p=0.3$

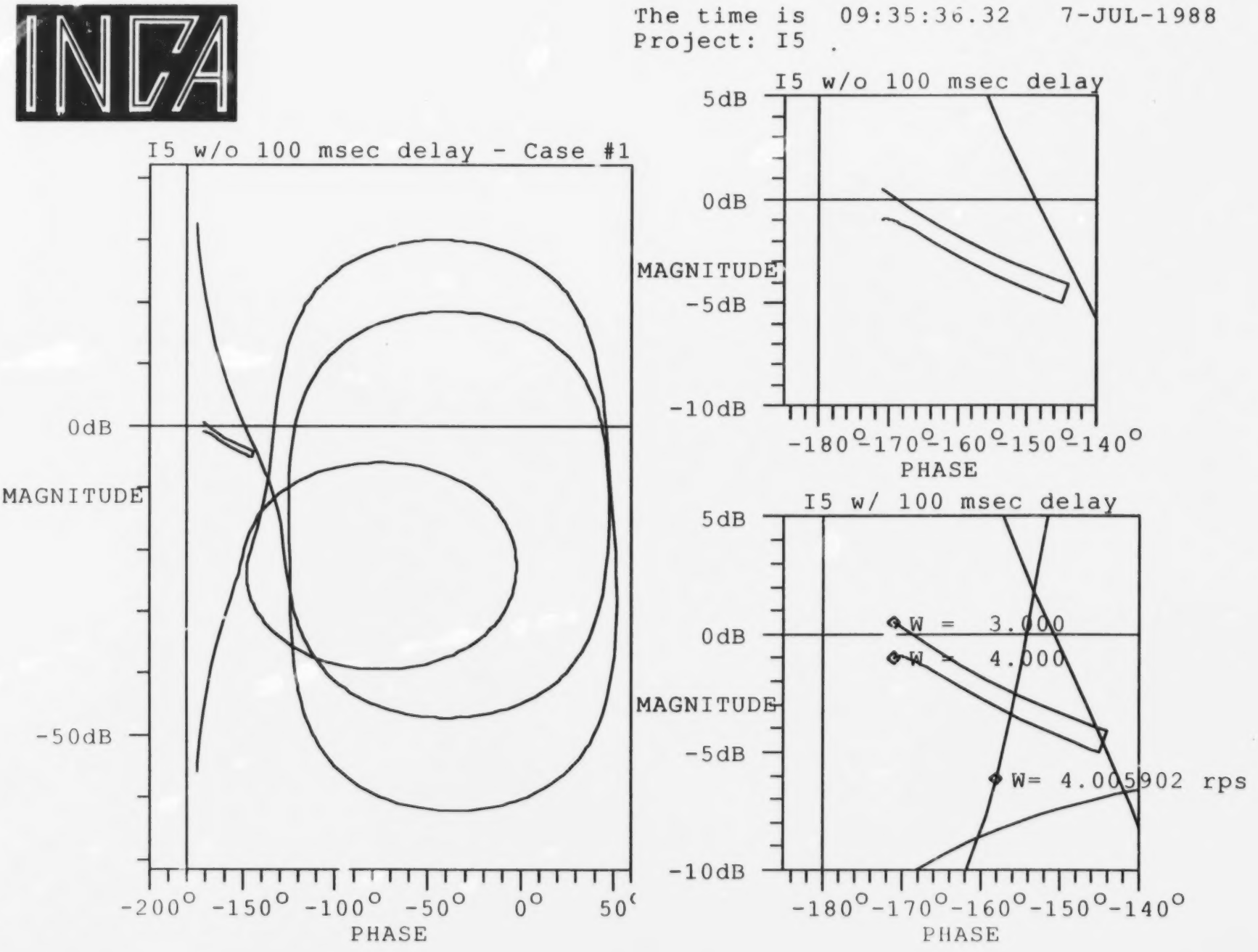
Case #4: Lead-Lag Compensator with Notch Filtering

- Case #1 with $z=3.4$, $\zeta_z=0.002$, $p=3.4$, $\zeta_p=1$

Case #5: Lead-Lag Compensator with Notch Filtering

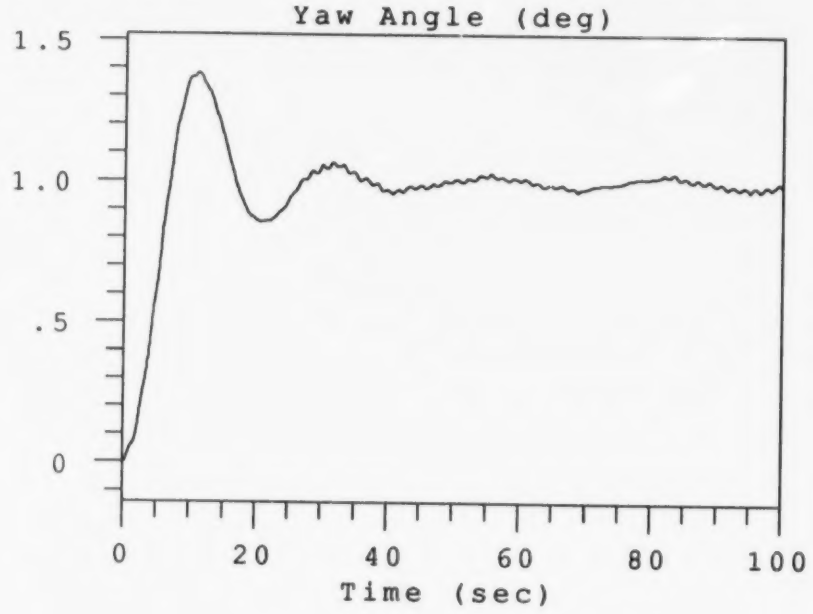
- Case #1 with $z=3.6$, $\zeta_z=0.002$, $p=3.6$, $\zeta_p=1$

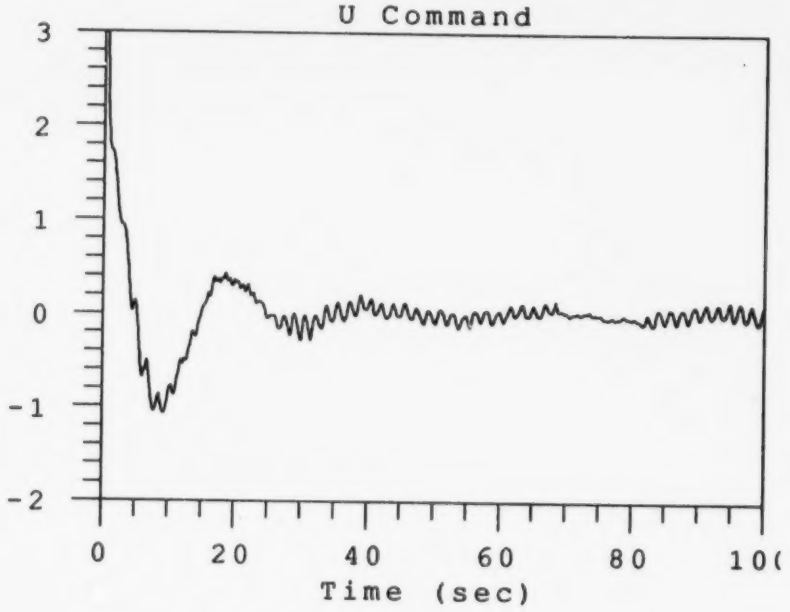
The left plot displayed on the opposing page shows the small nonlinear gain and phase margins of case #1 with respect to strucural mode limit cycling. The plot on the top right is close-up view of the area in the vicinity of the describing function. The plot on the bottom right side of the page is also a close-up view of case #1 with 100 msec time delay; structural mode limit cycling at 3.5 rad/sec is predicted from describing function analysis.

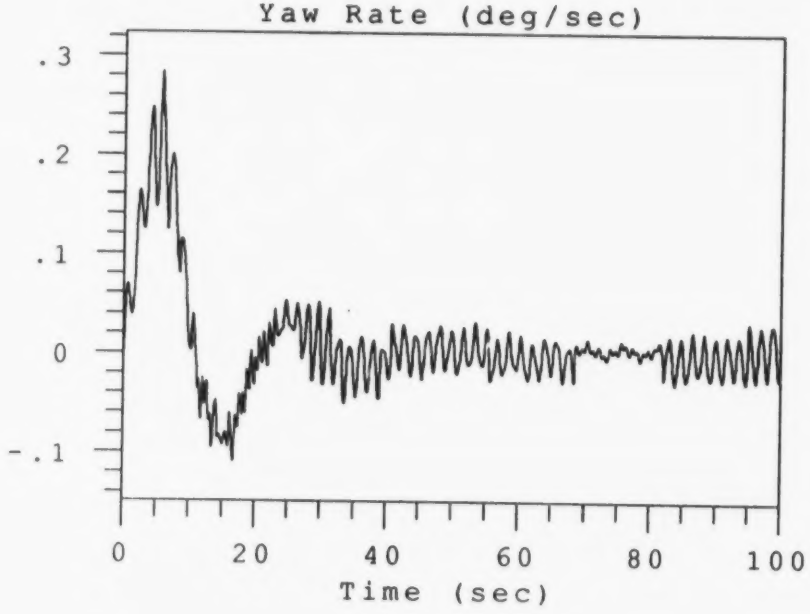

The time is 09:35:36.32 7-JUL-1988
Project: I5 .
I5 w/o 100 msec delay - Case #1
0dB
MAGNITUDE
-50dB
-200° -150° -100° -50° 0° 50°
PHASE
I5 w/o 100 msec delay
5dB
0dB
MAGNITUDE
-5dB
-10dB
-180°-170°-160°-150°-140°
PHASE
I5 w/ 100 msec delay
5dB
0dB
MAGNITUDE
-5dB
-10dB
W = 3.000
W = 4.000
W= 4.005902 rps
-180°-170°-160°-150°-140°
PHASE

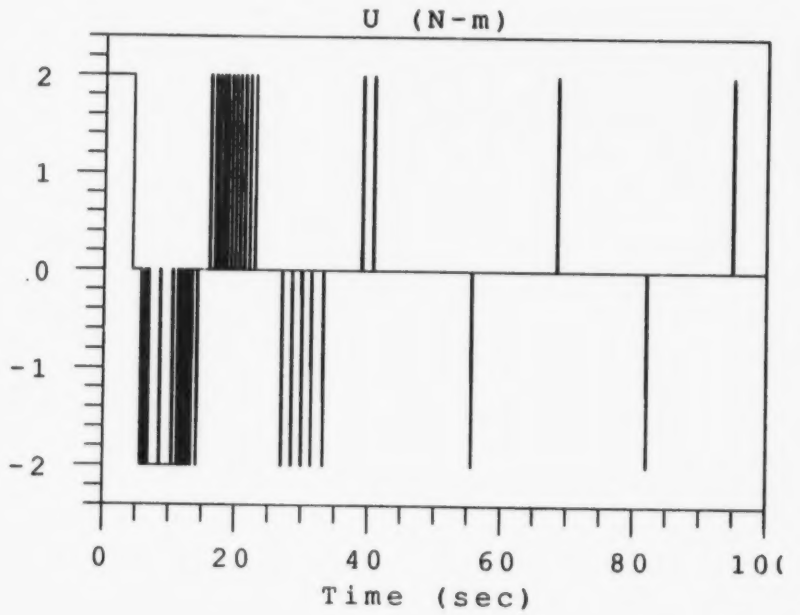
These plots represent the nonlinear simulation of case #1 without delay; note the absence of limit cycling in the plot on the lower right.

File: WORM.DAT Case #1 - no delay The time is 11:01:33.49 10- UL-1988



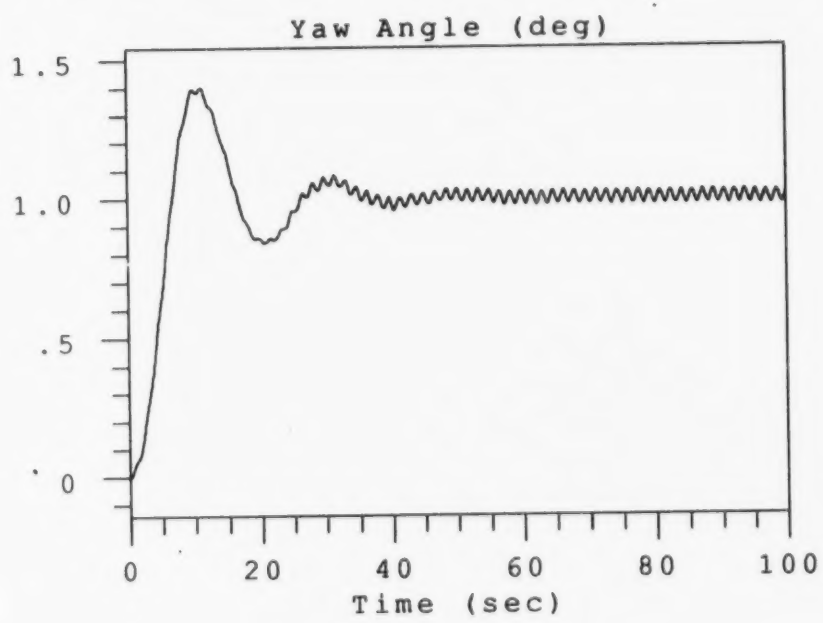


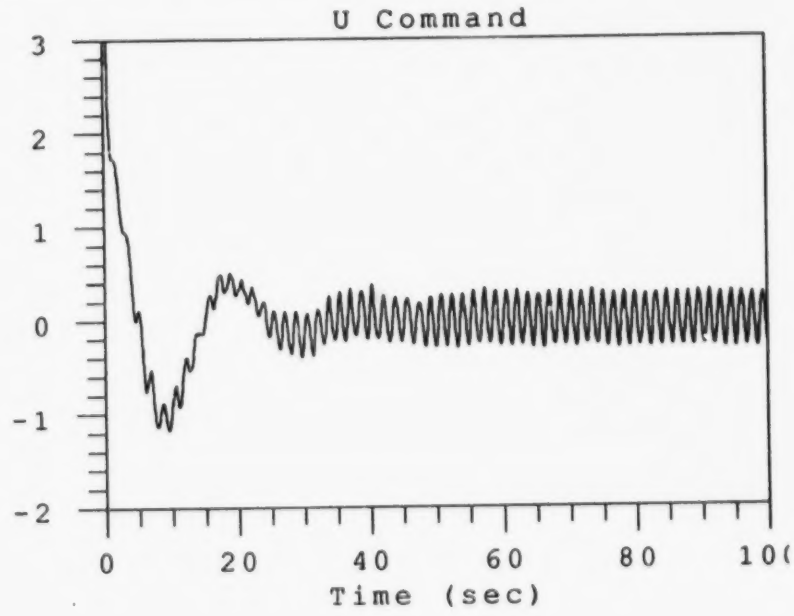


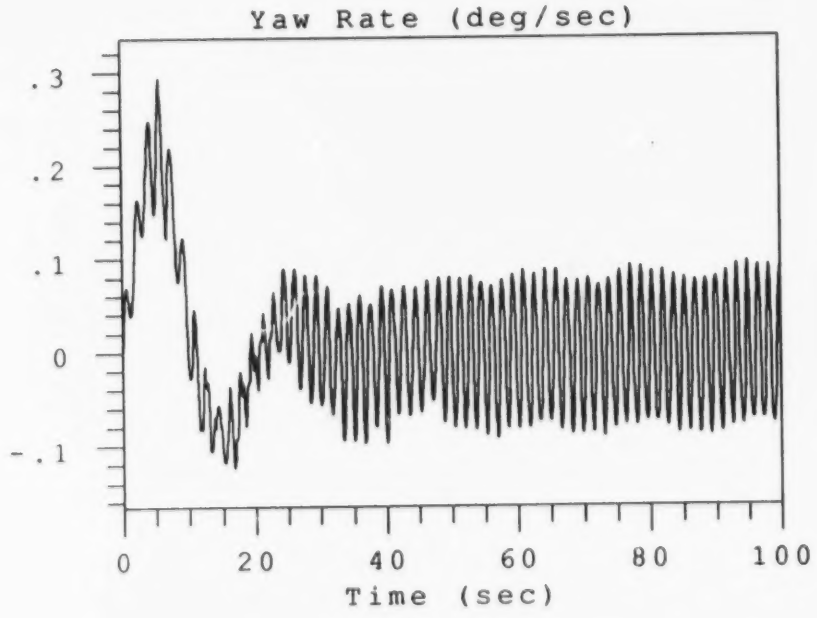


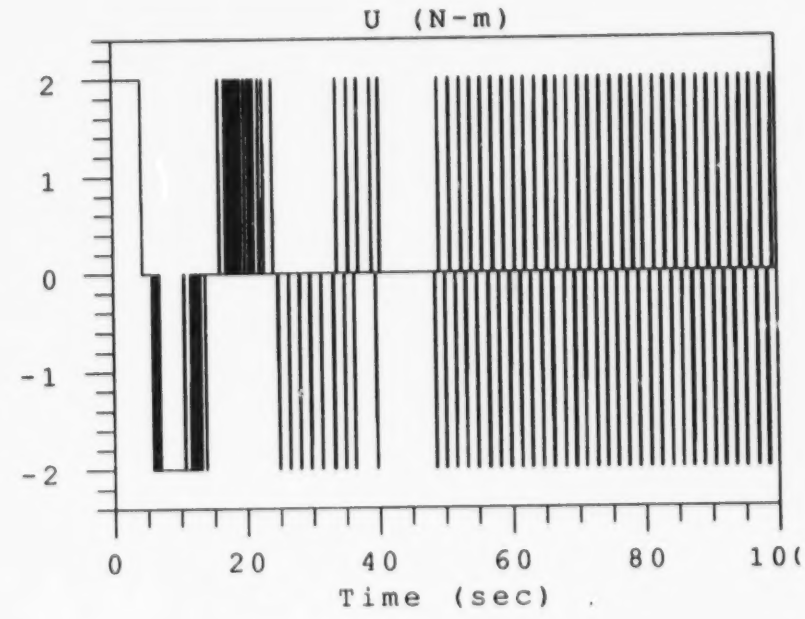
These plots represent the nonlinear simulation of case #1 with delay; note the appearance of limit cycling in the plot on the lower right.

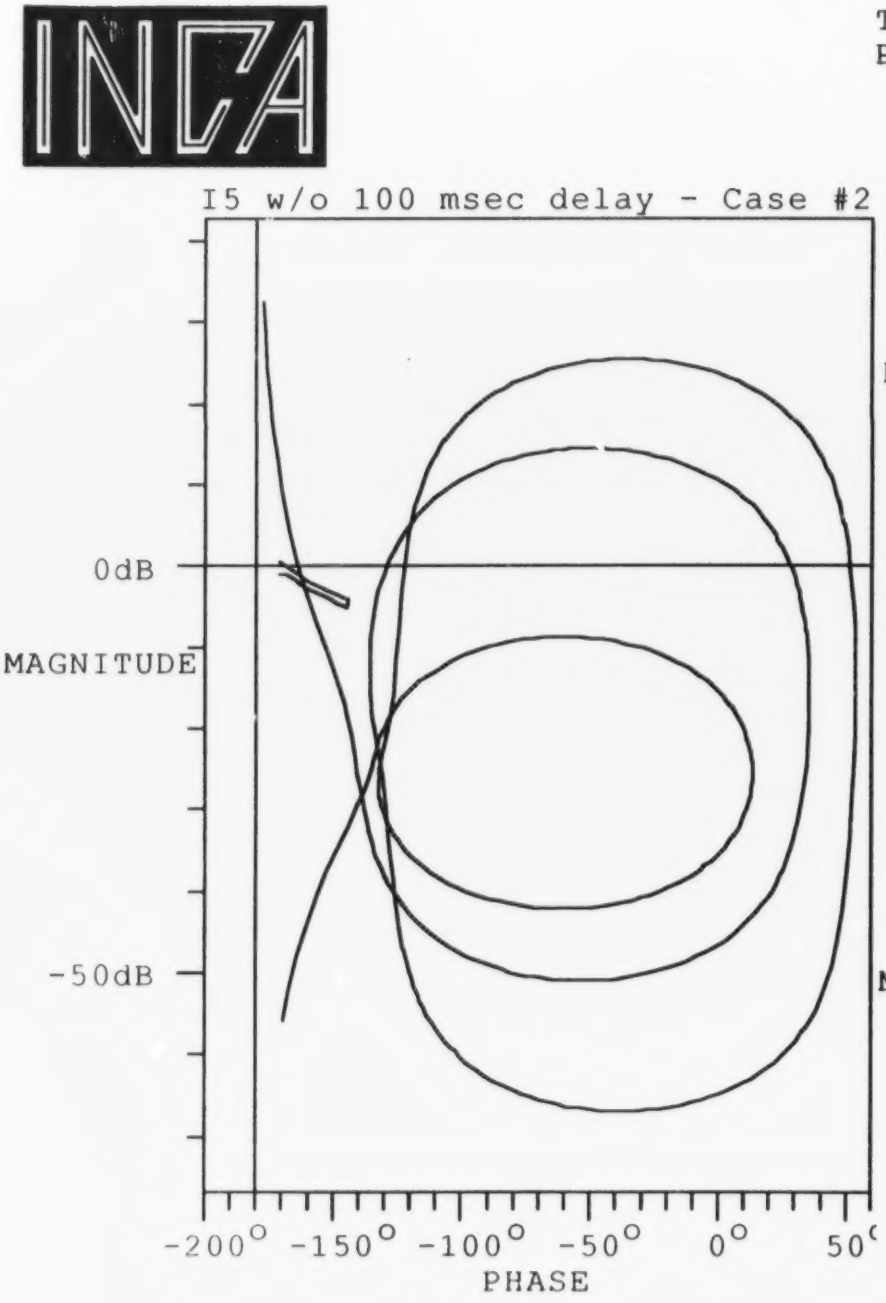
File: WORM.DAT Case #1 with delay The time is 11:12:50.83 10-JUL-1988



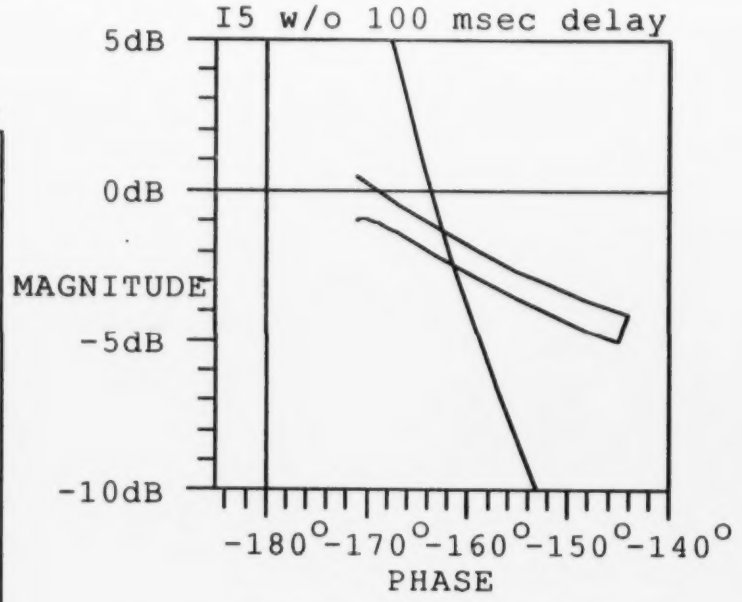


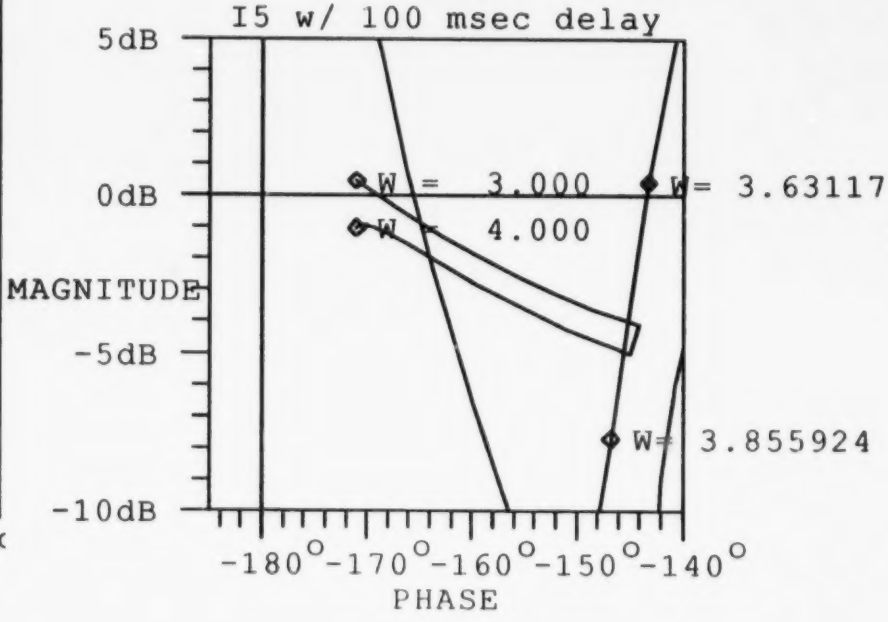






The time is 09:54:03.04 7-JUL-1988
Project: I5





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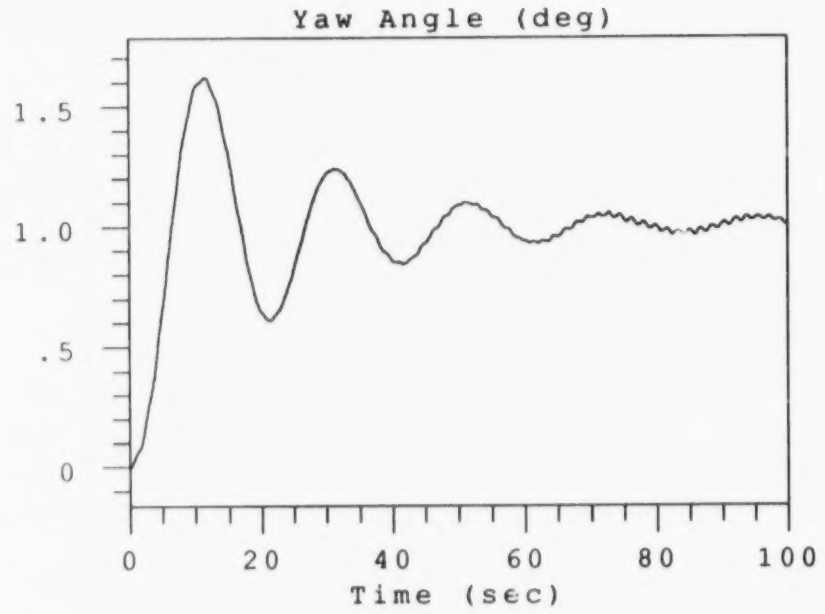

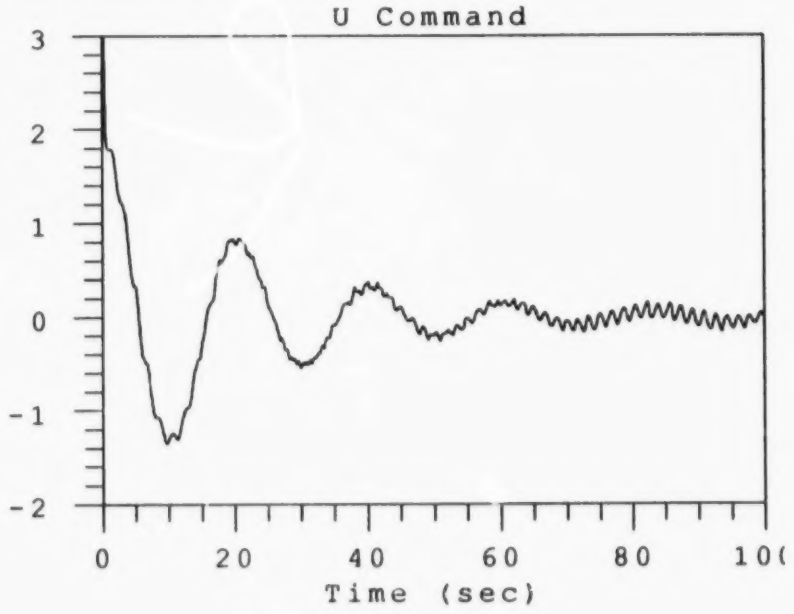

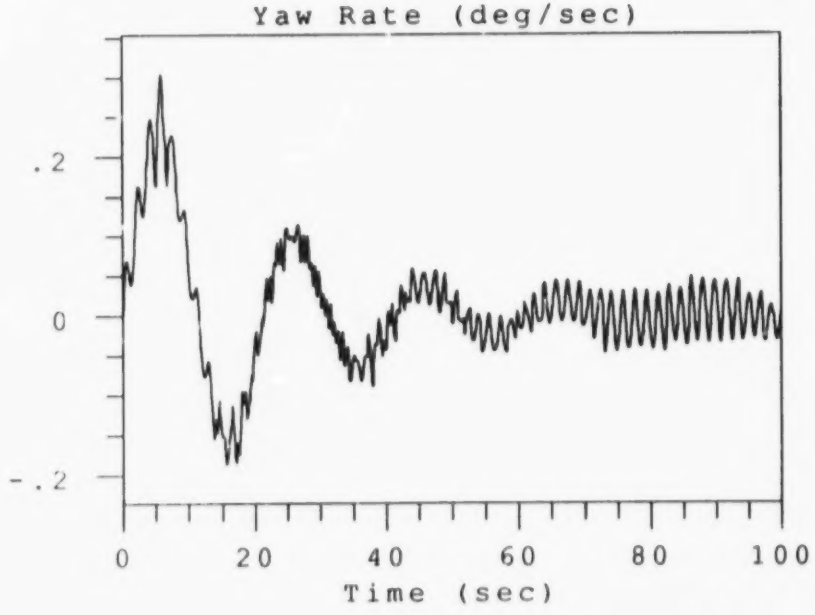

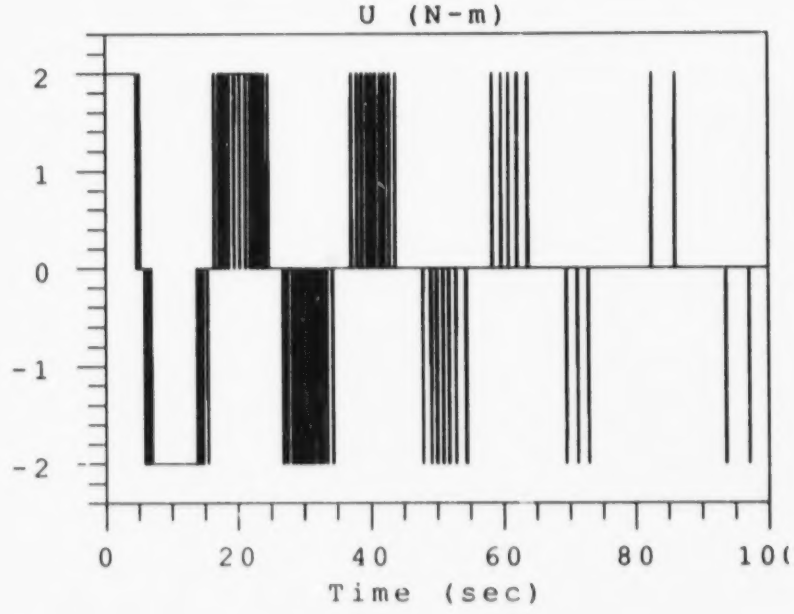

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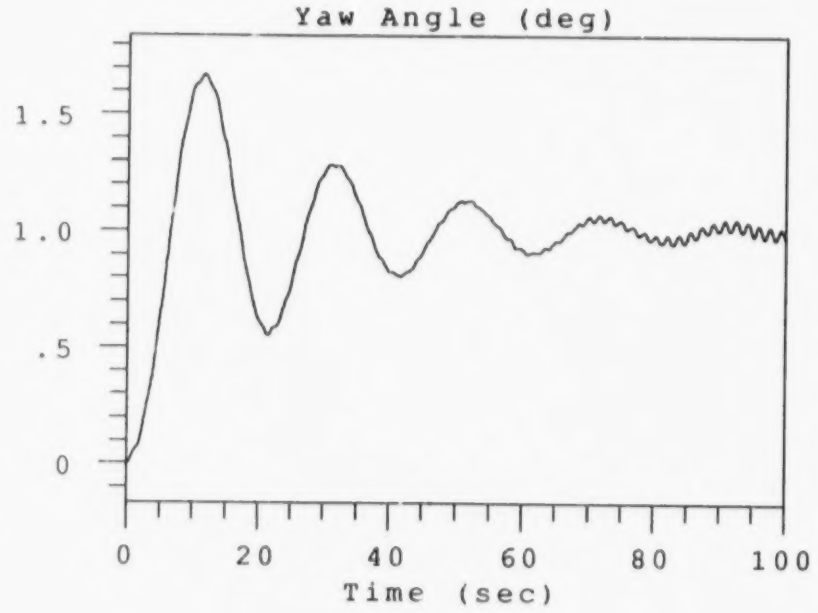

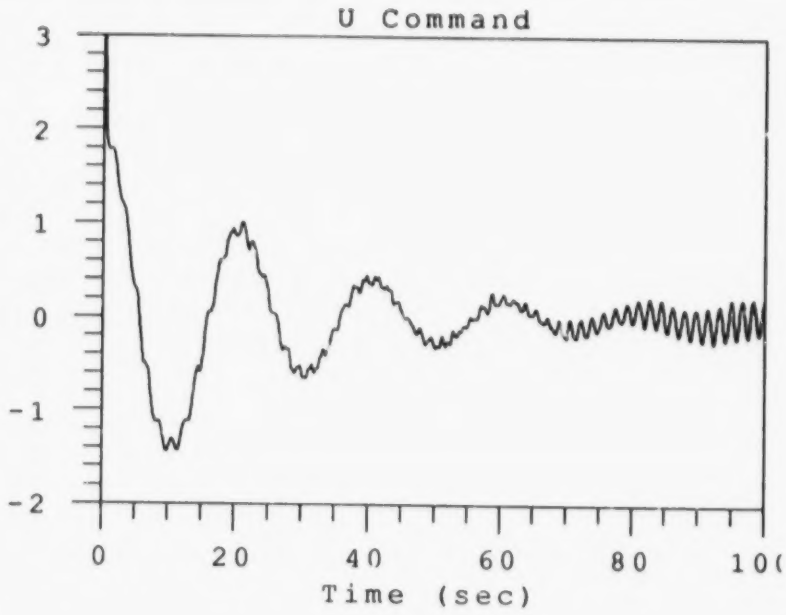

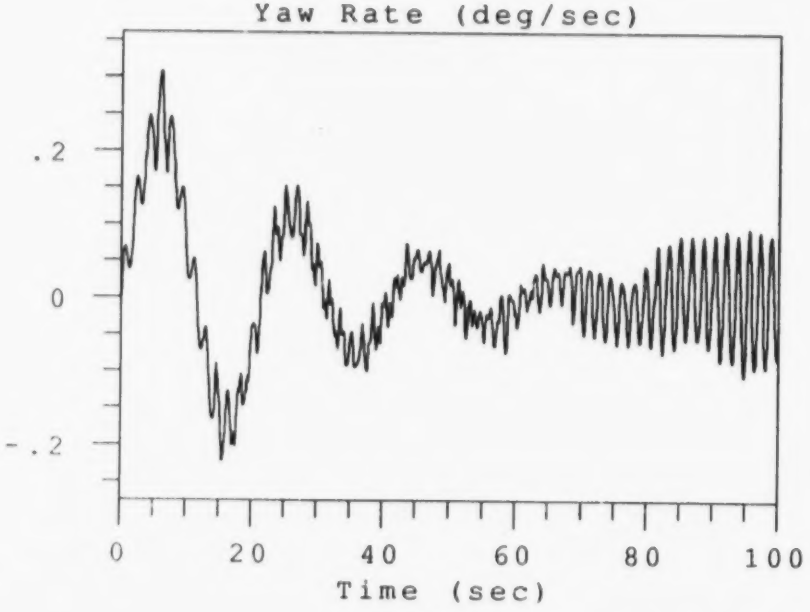

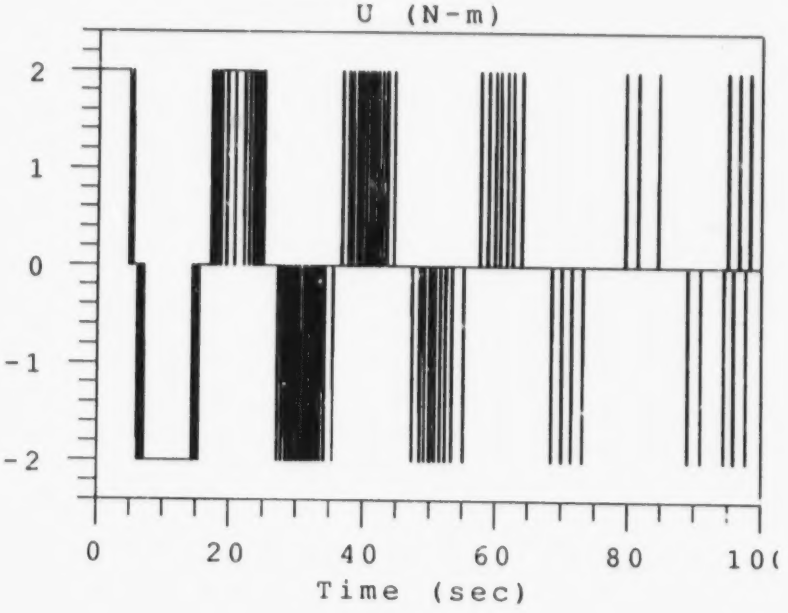

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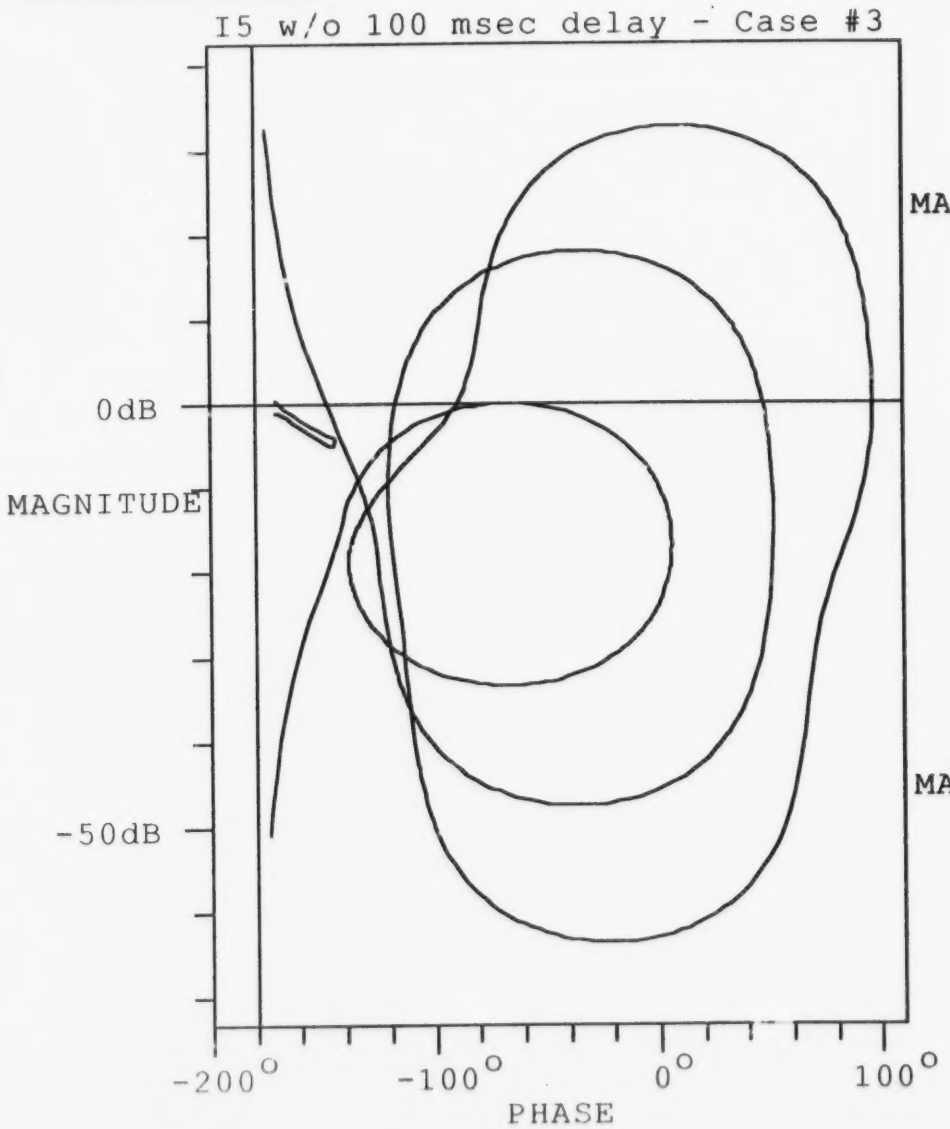
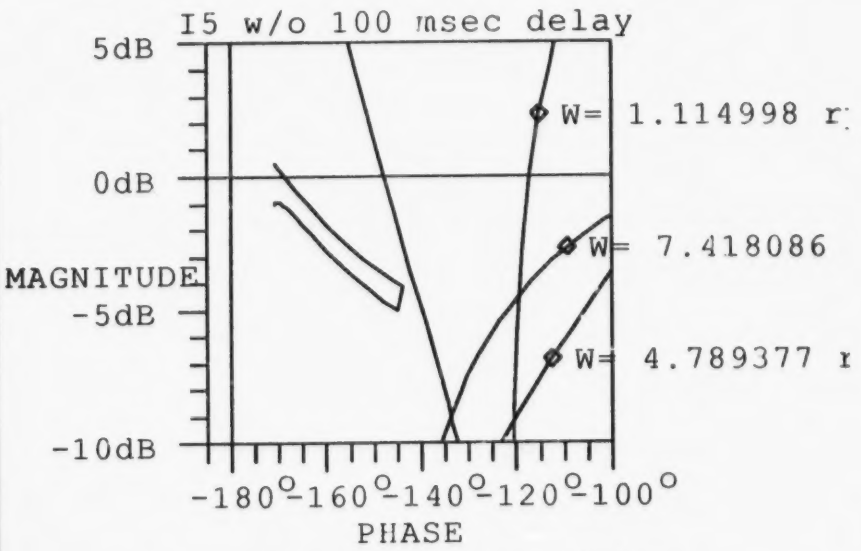
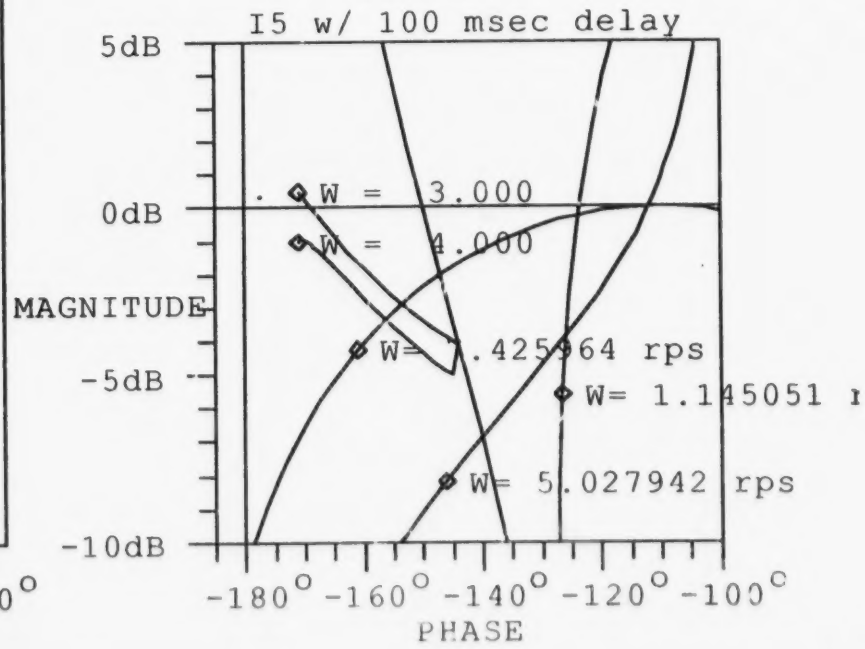
Project: I5







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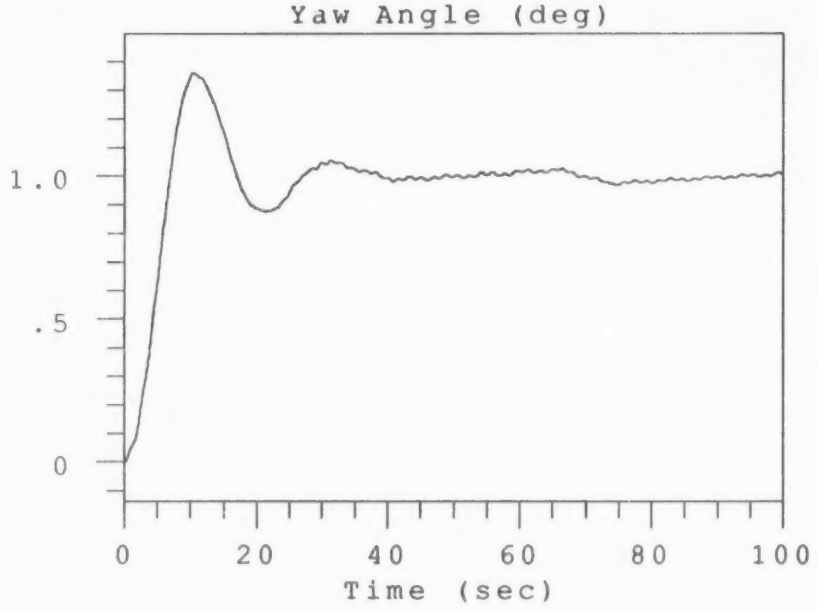

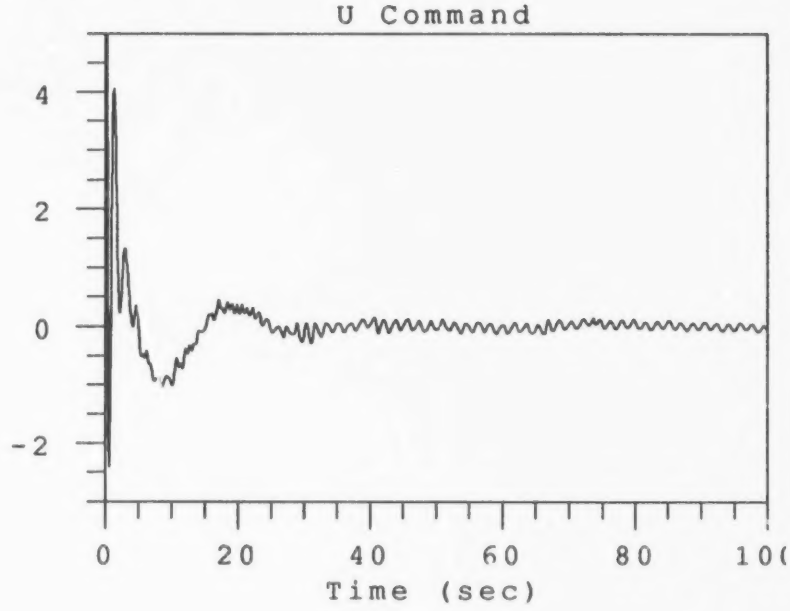

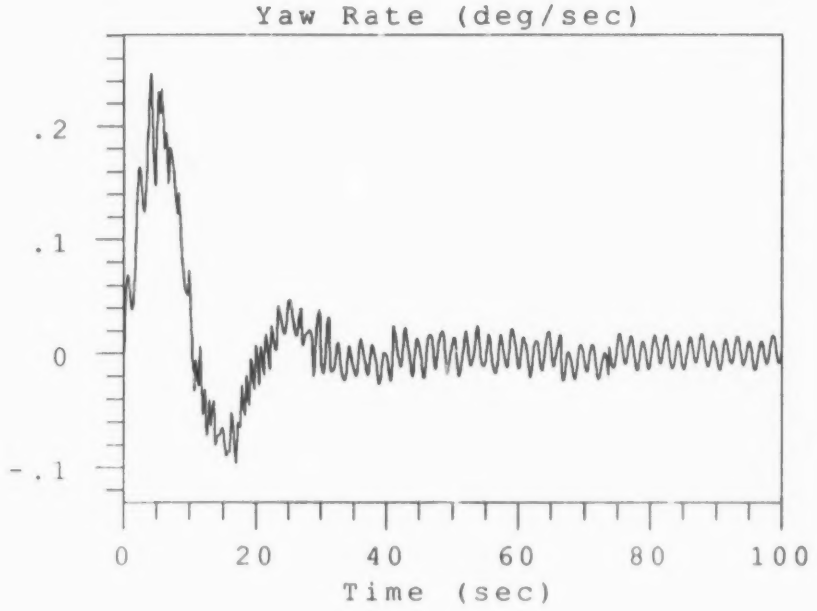

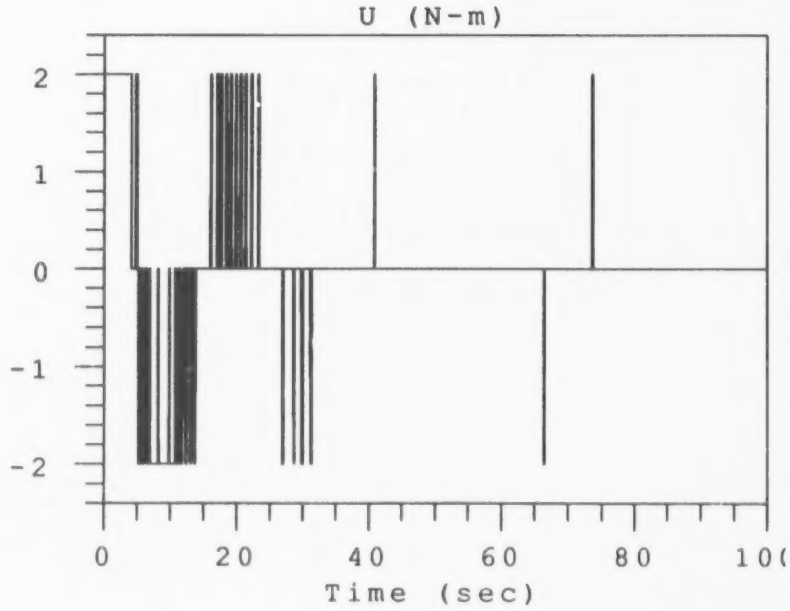

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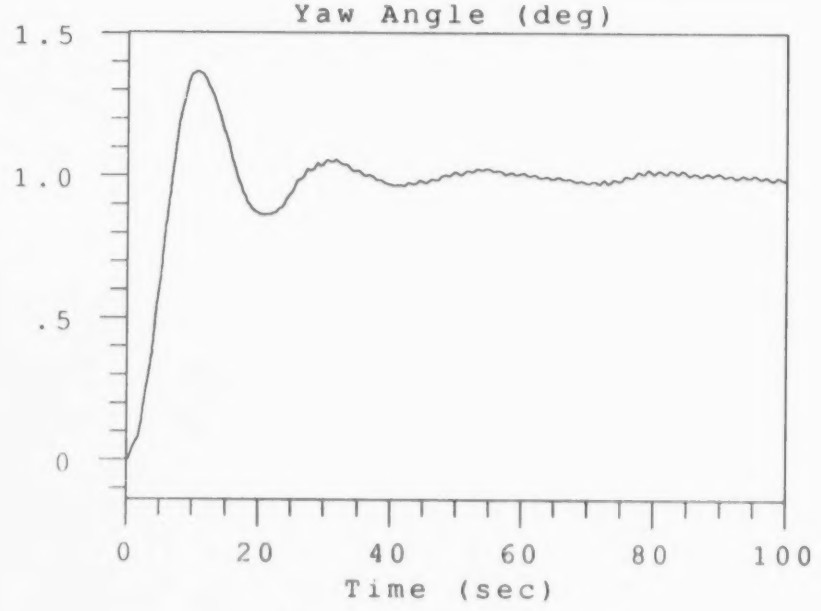
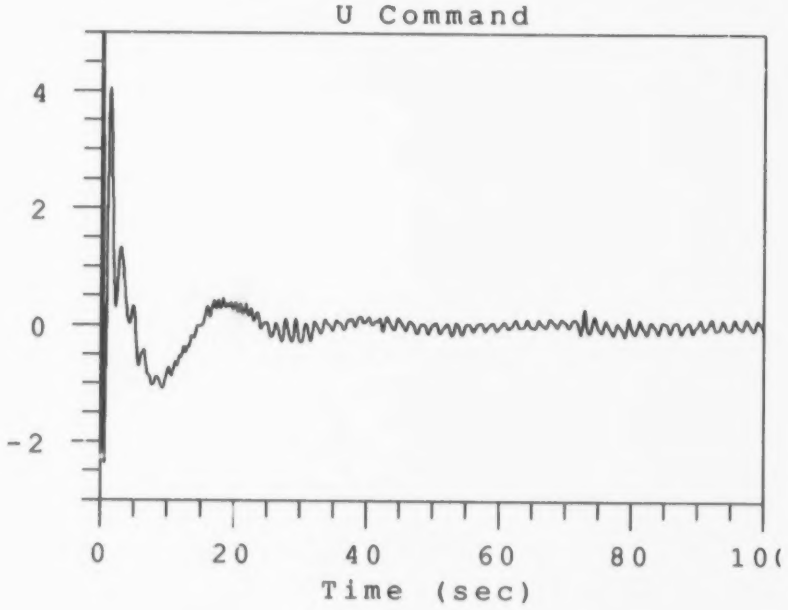
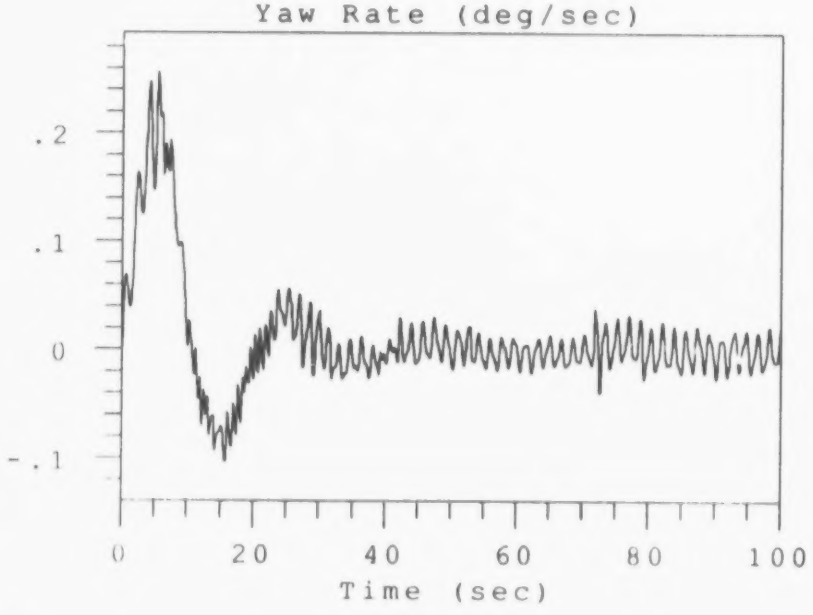
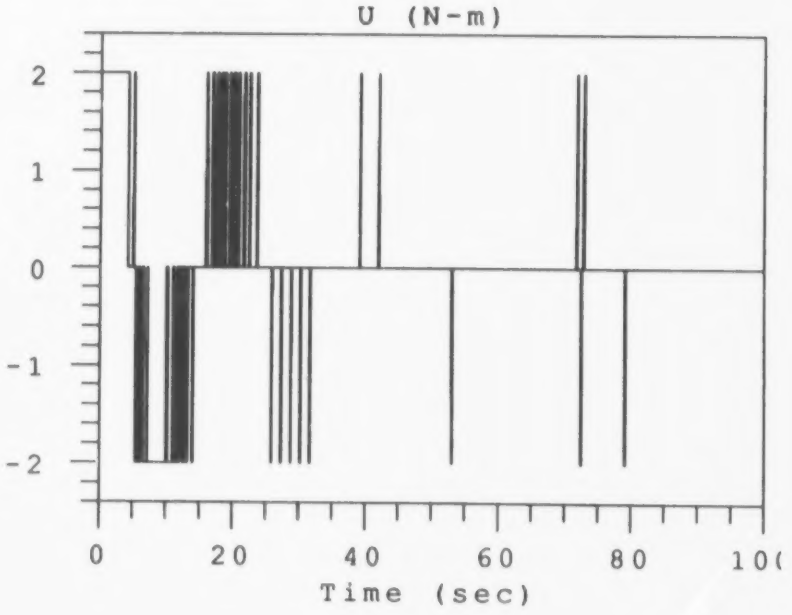
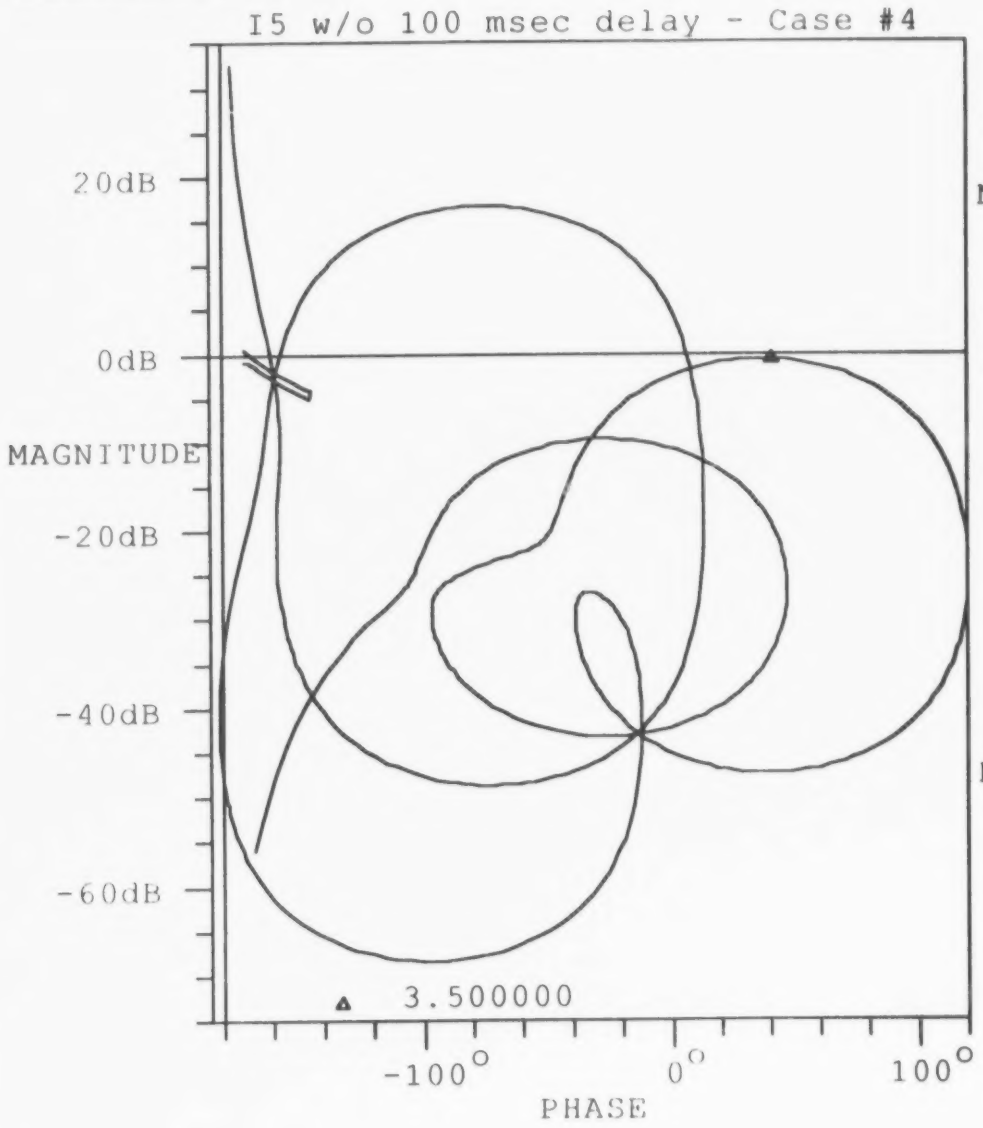


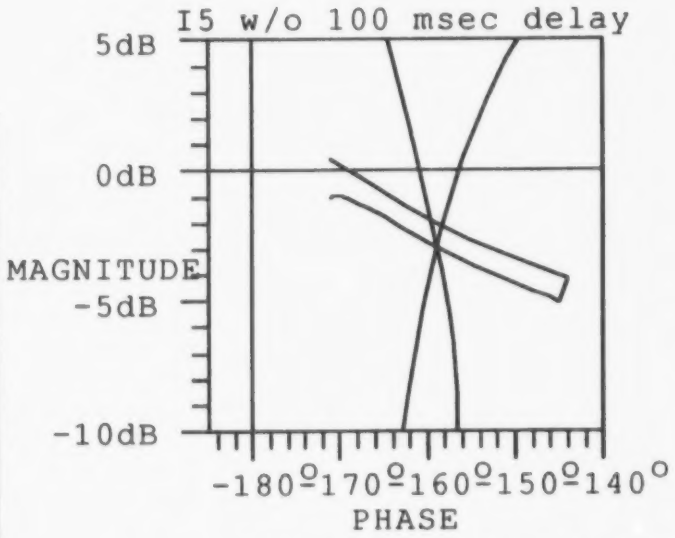


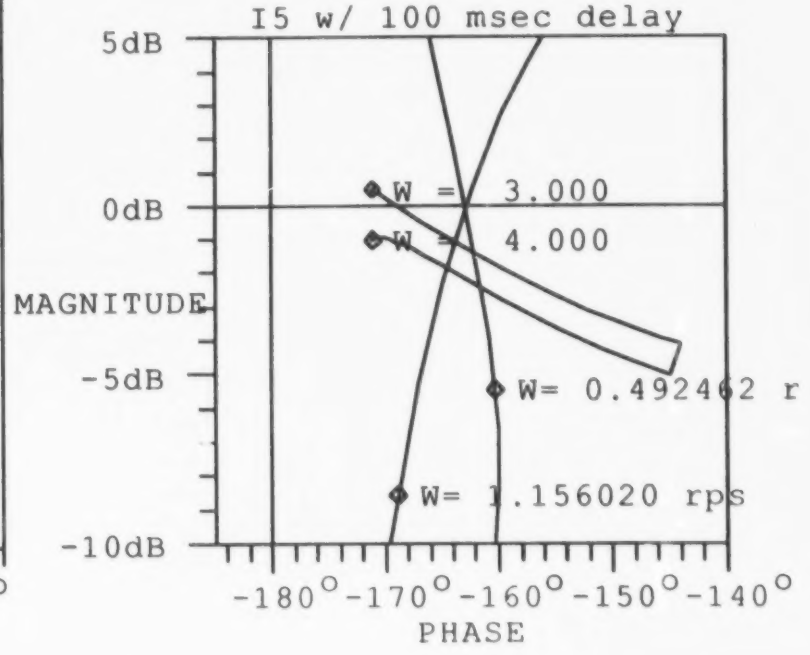




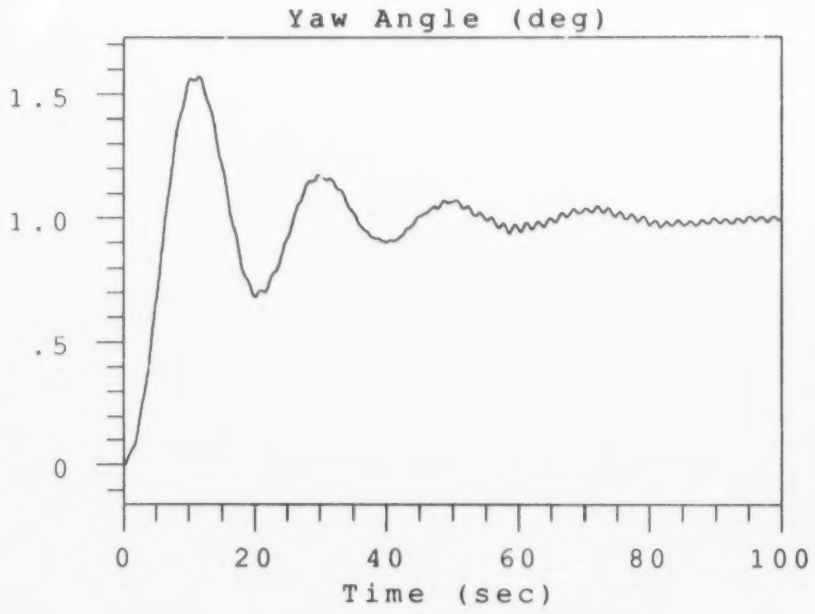
The time is 09:01:12.72 7-JUL-1988
Project: I5

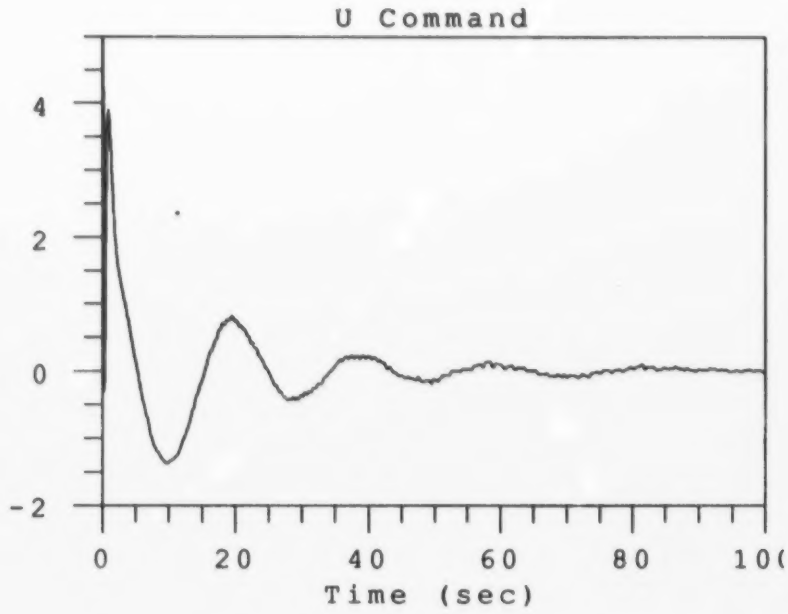


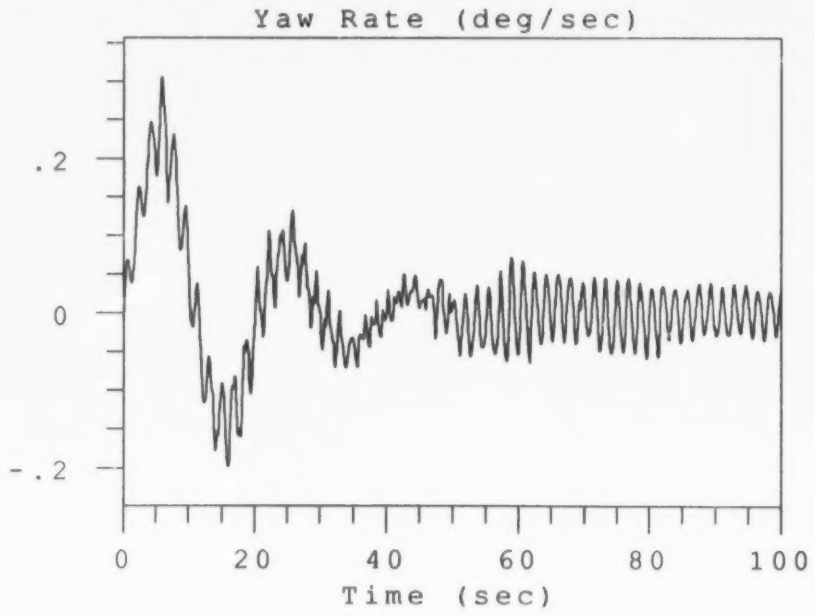


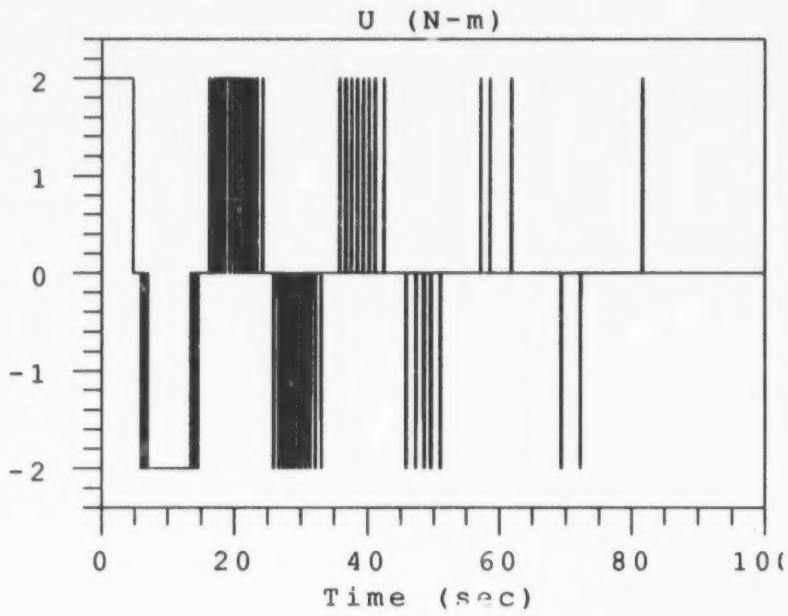


File: WORM.DAT Case #4 no delay The time is 12:18:52.36 10-JUL-1988









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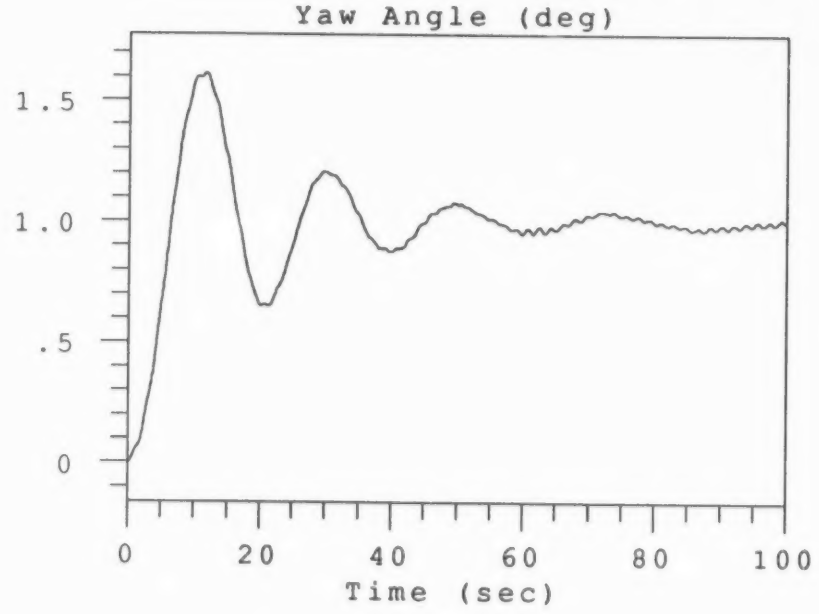
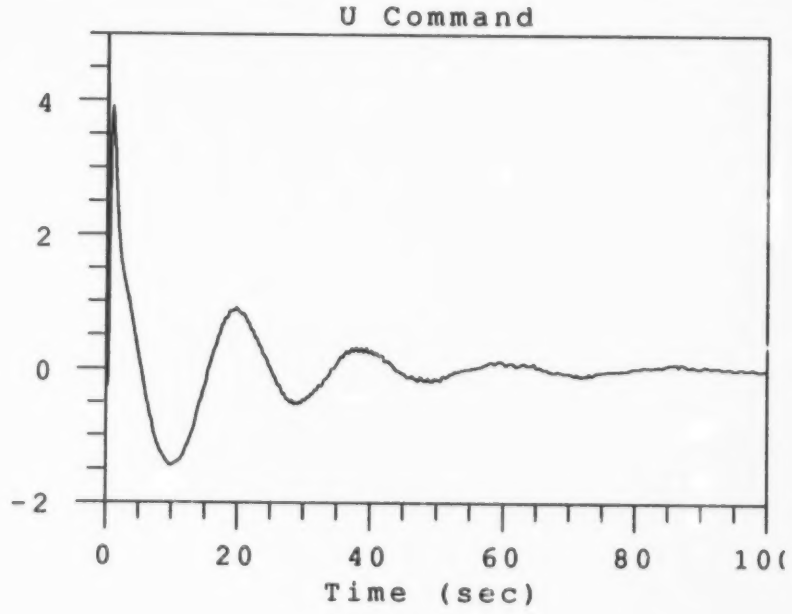
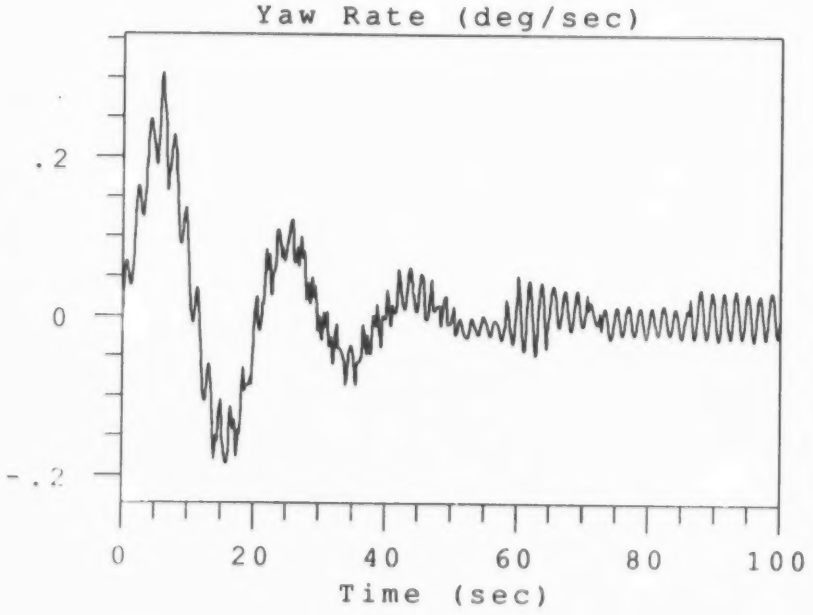
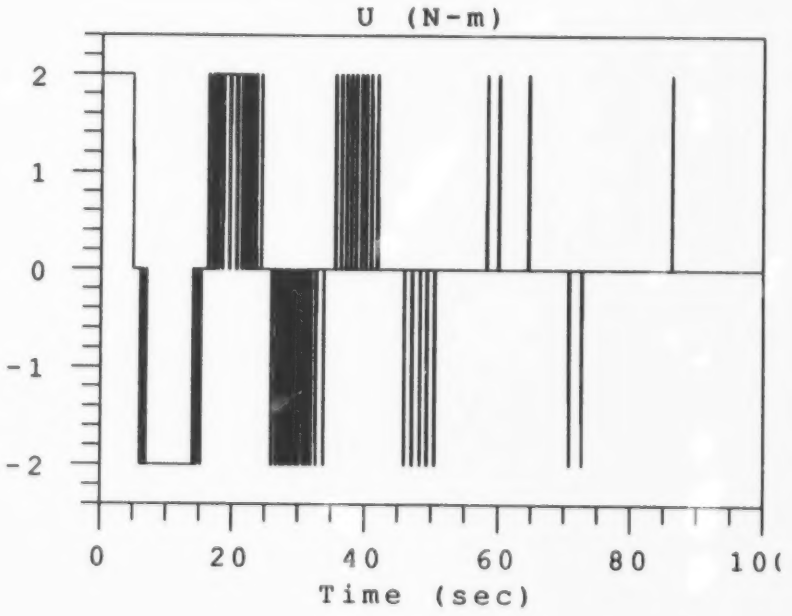
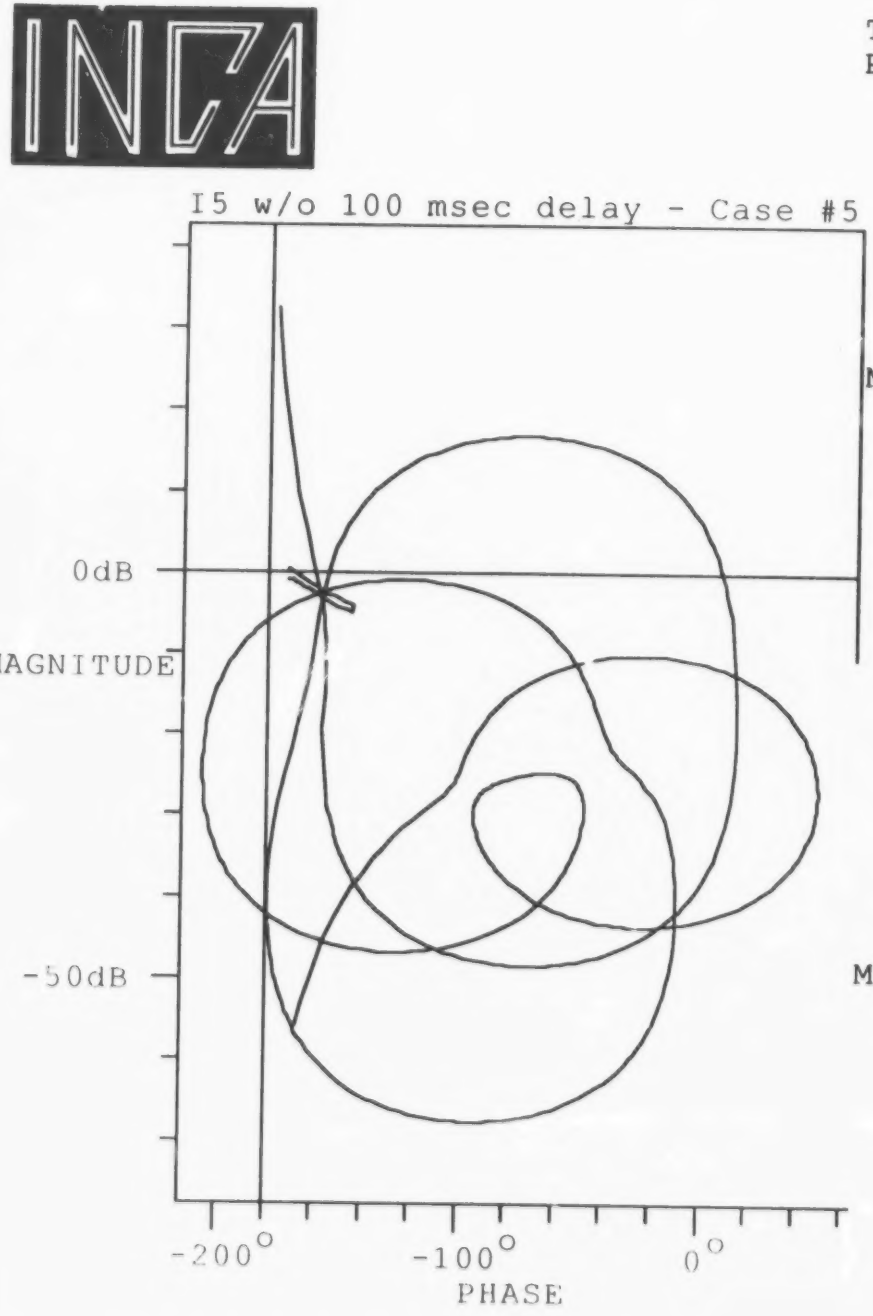


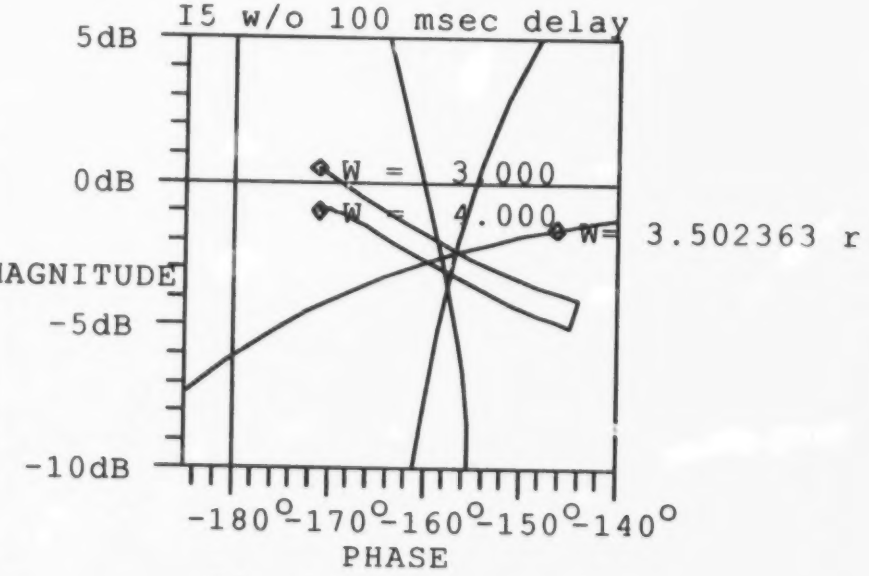


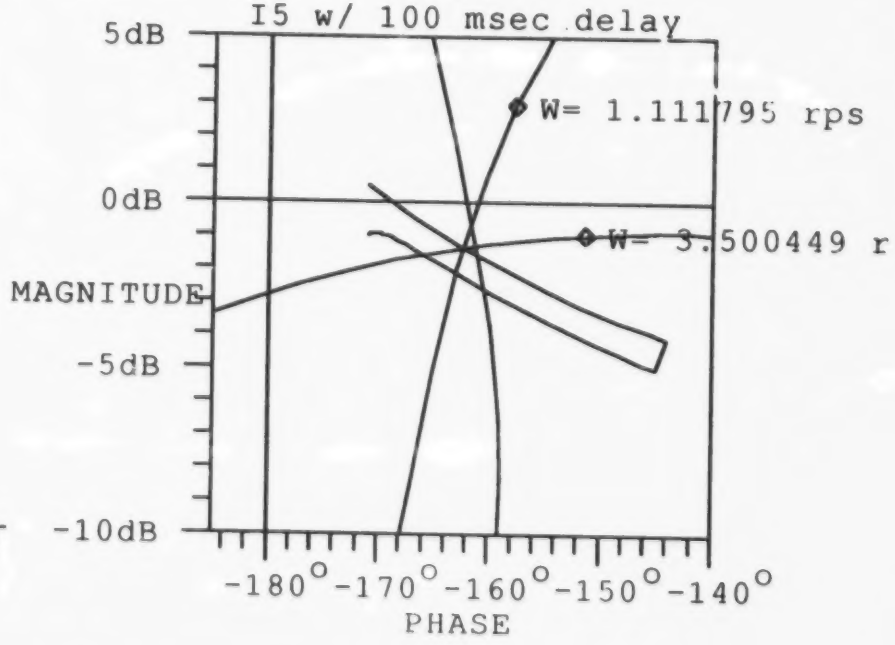




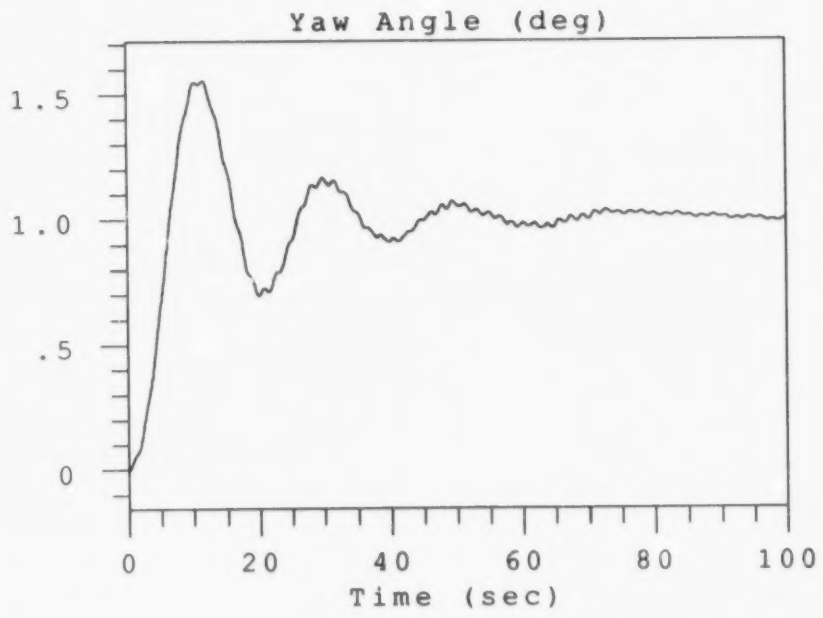


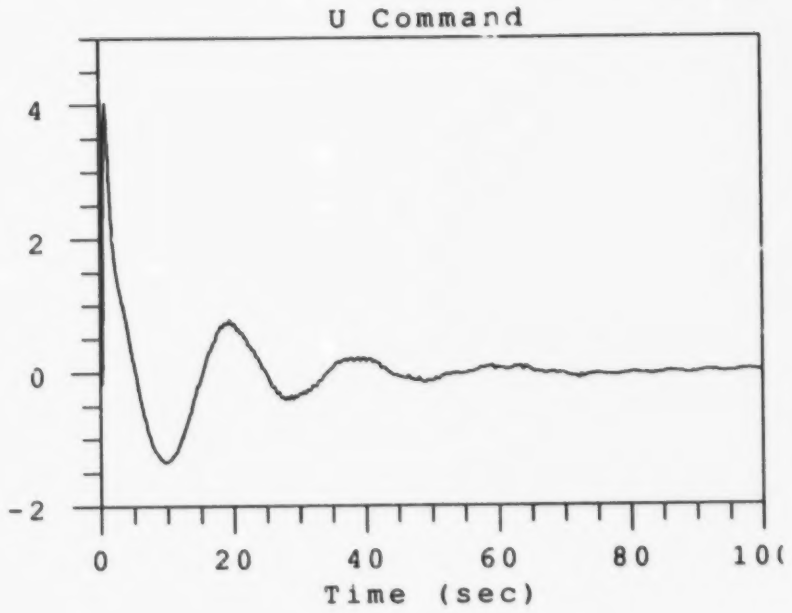
The time is 09:20:31.68 7-JUL-1988
Project: I5

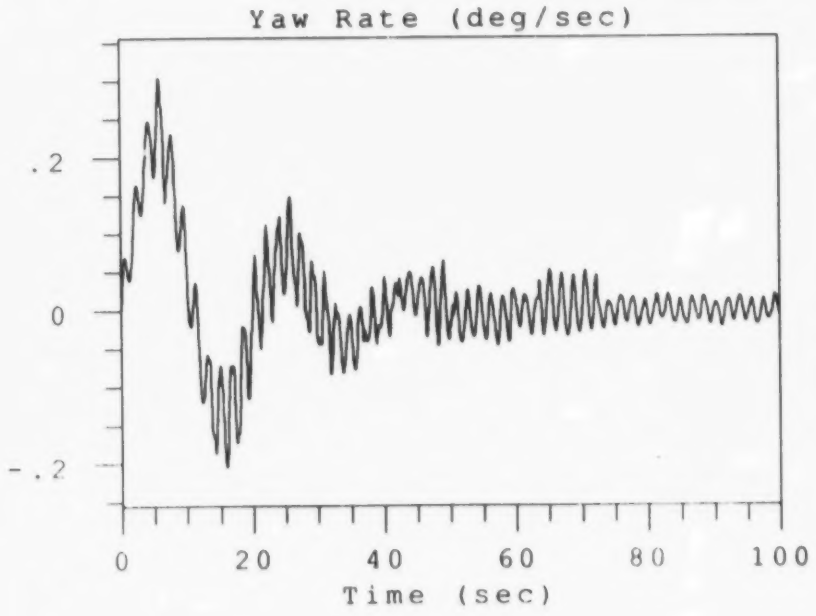


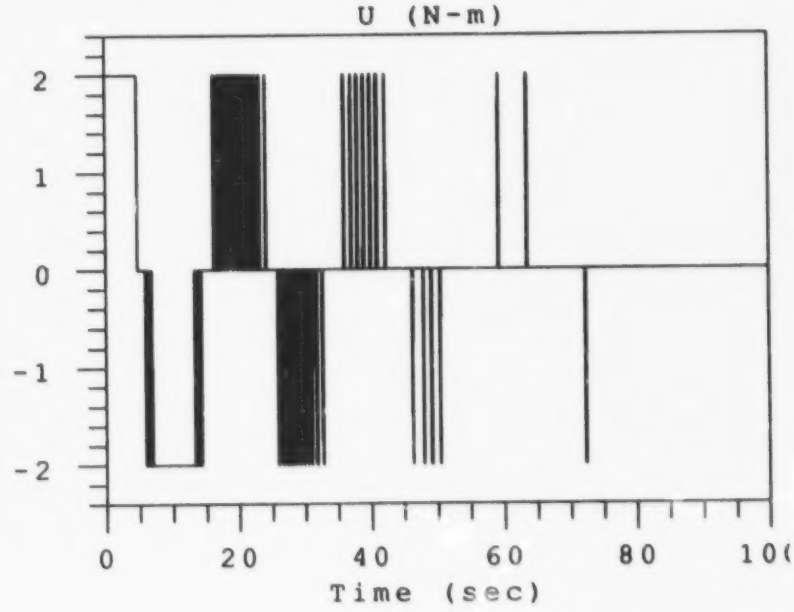


File: WORM.DAT Case #5 no delay The time is 12:45:47.79 10-JUL-1988









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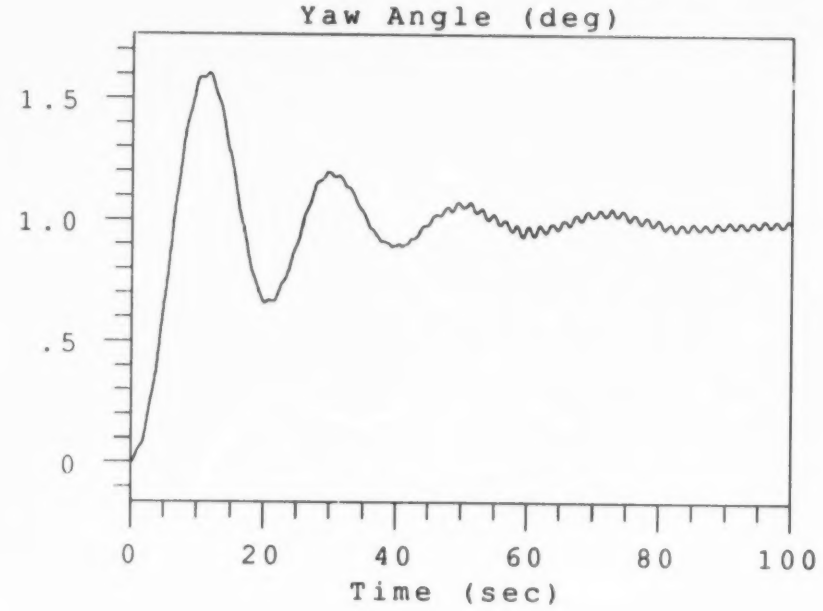

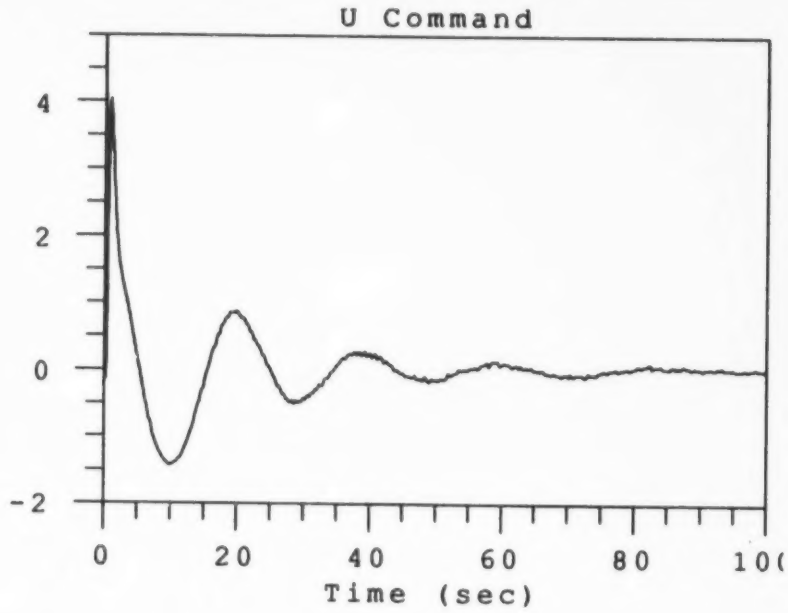

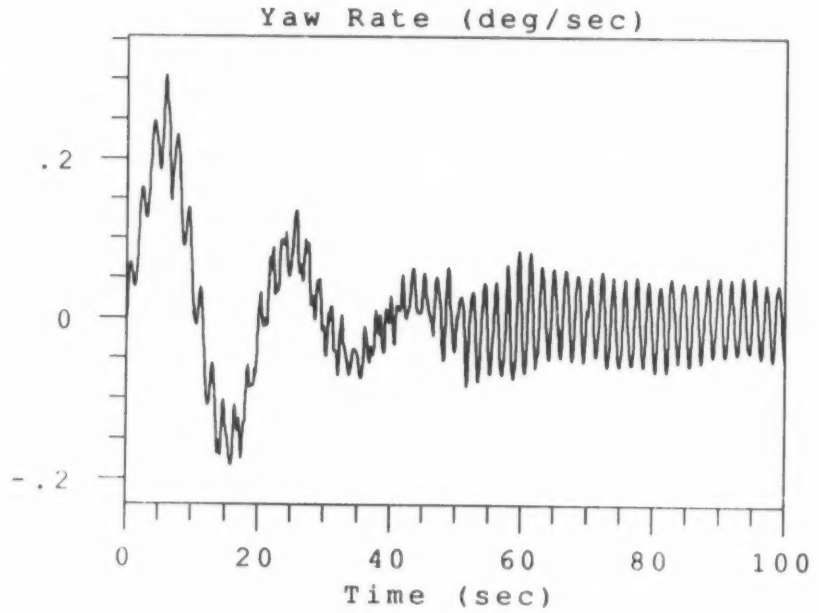

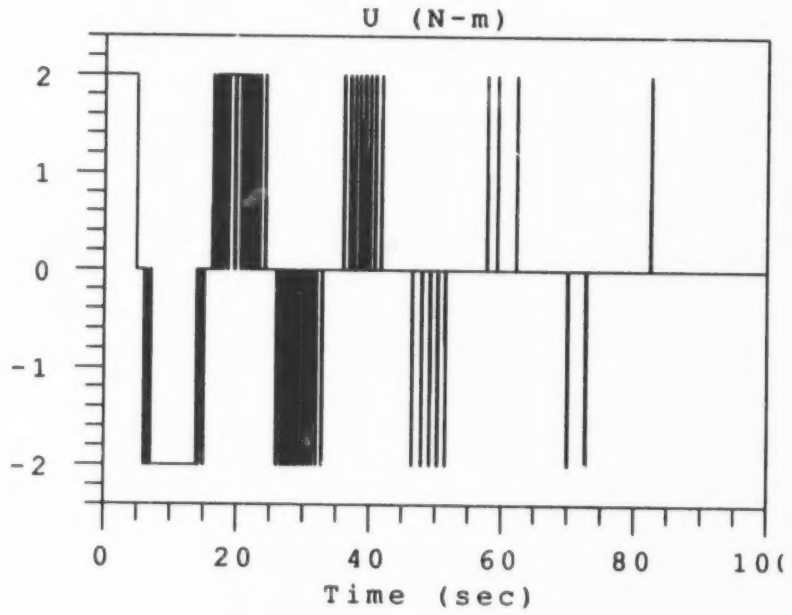

Contents

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  - Project schedule
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- Introduction to INCA program
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  - Nonlinear control elements
  - Single-input describing function (SIDF)
  - INCA enhancement for SIDF analysis
- Task 2: Nonlinear control synthesis using SIDF
- Plan for the second year: Tasks 3 & 4
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- Appendix

Although the SIDF method is a useful tool for certain nonlinear systems, the so-called DIDF (Dual-Input Describing Function) method is needed in certain cases where two sinusoidal signals appear at the input of a nonlinear element. In these cases, the high frequency signal is taken to be the system limit cycle, while the slowly varying signal is coming from input commands or disturbances to the system. The concept of the DIDF is rather straightforward, but the application is often time consuming. Tasks 3 & 4 will involve the implementation of the DIDF method to the INCA program.

Plan for the Second Year (Tasks 3 &4)

- DIDF - Used where input signal to nonlinearity consists of a bias and a sinusoid.

- Applicable to situations where a constant disturbance torque is present, e.g., during stationkeeping or reboost maneuvers.

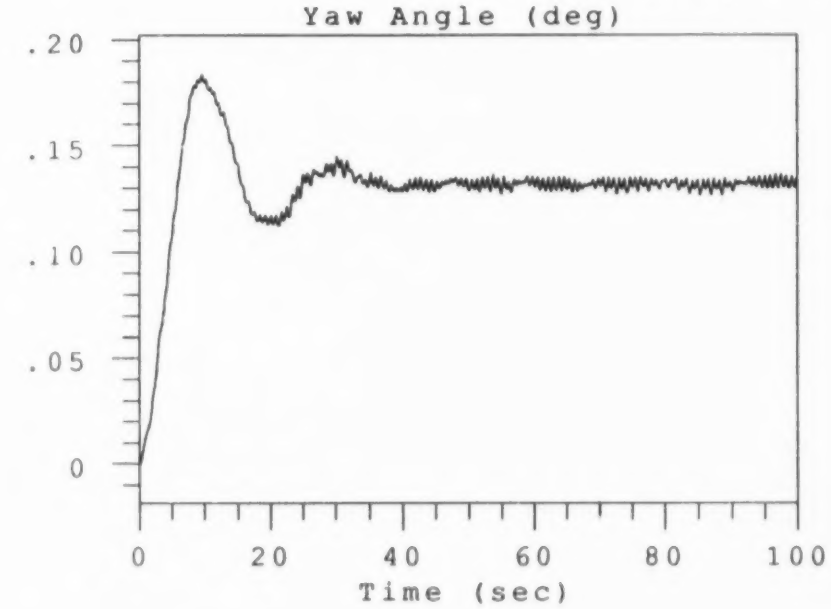
- Second-year work includes:
  - Addition of DIDF algorithms for existing INCA nonlinearities.
  - Development of DIDF synthesis methodology.

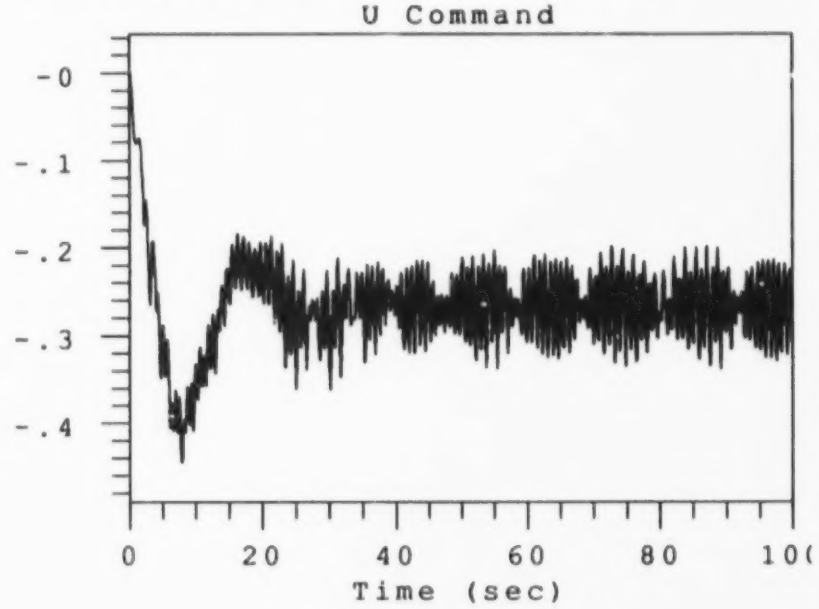
The plots on the opposing page represent a variation of the cases presented earlier in the report. The example presented here depicts the INTELSAT 5 spacecraft with the lead-lag filter used in case #1; in this example, a constant torque of 0.5 N-m. is introduced into the system. Note the steady-state value of yaw angle with the "beating" effect. More importantly, notice the lack of limit cycling in the torque output plot. The thruster is counteracting the 0.5 N-m torque input with continuous firing; because the disturbance torque is one-quarter of the thruster output, a 25% duty cycle in the thruster firing is evident. The absence of a limit cycle is due to the steady-state negative bias of the torque command visible in the top-left plot. The input command activates the negative portion of the PWPF modulator and as a result, no positive torque pulse is possible.

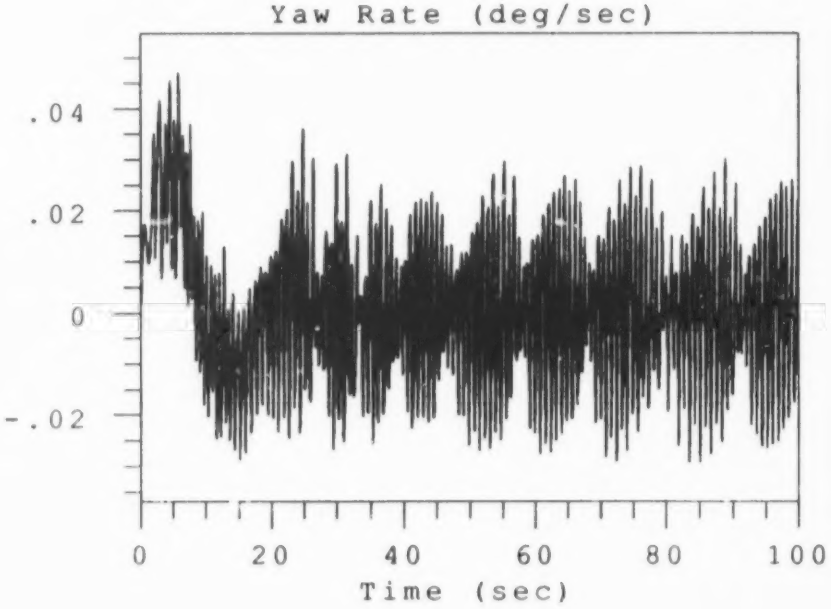
This system's behavior can be predicted by the DIDF method. The SIDF describing function method is no longer applicable because the input to the modulator is no longer a pure sinusoid; the torque disturbance introduces a bias with the sinusoid. Therefore, the SIDF describing function method is no longer applicable.

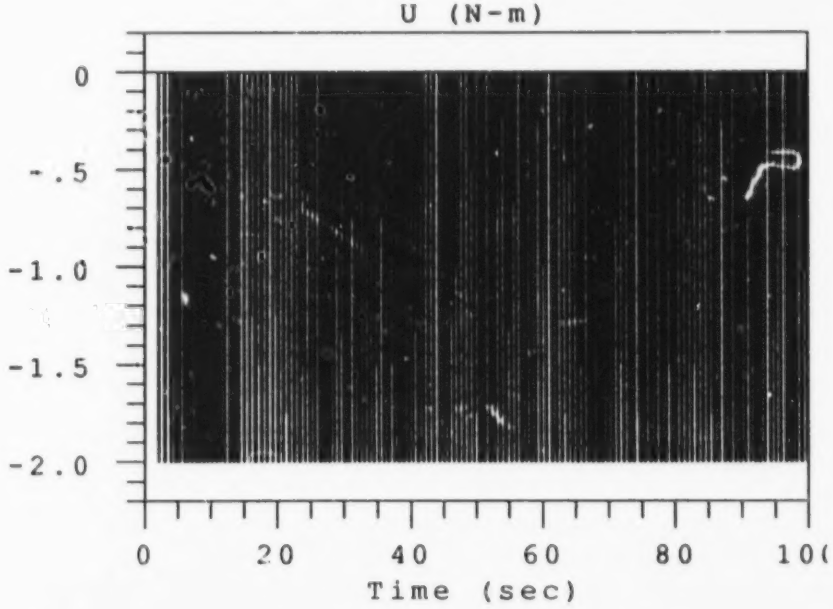
File: WORM.DAT The time is 13:03:24.63 10-JUL-1988

Case #1 w/ delay & 0.5 N-m disturbance





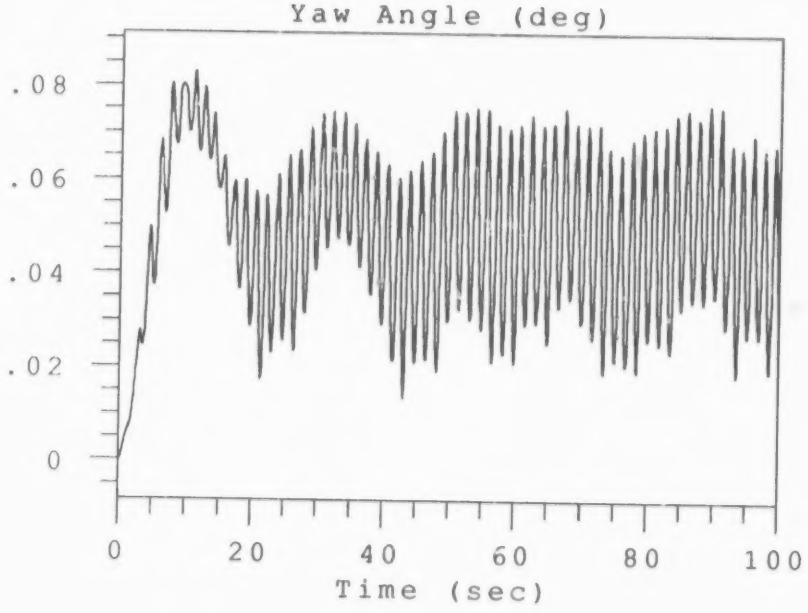


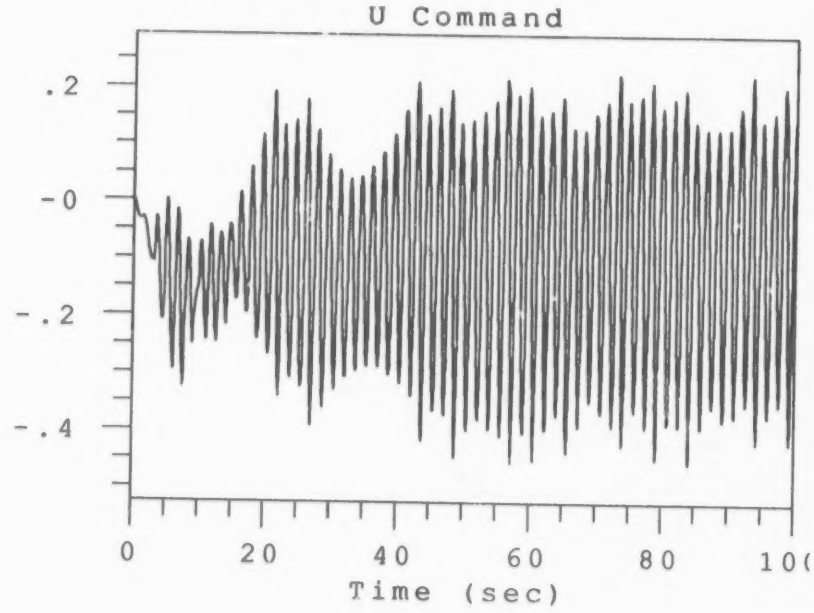


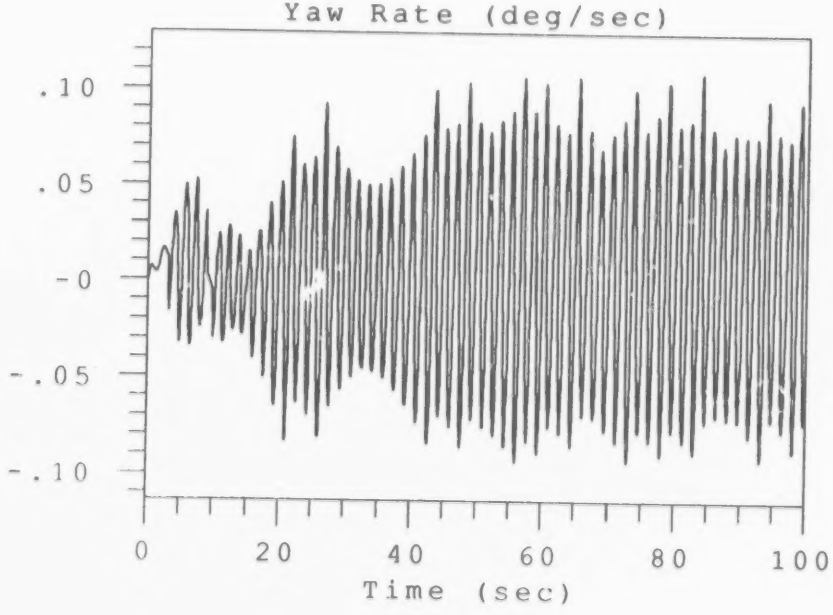
This system is nearly identical with the one of the previous page except that a torque bias of 0.2 N-m is introduced as indicated by the lower duty cycle firing of 10%. The torque command has a slightly negative average bias; as a result, some positive thrusting occurs.

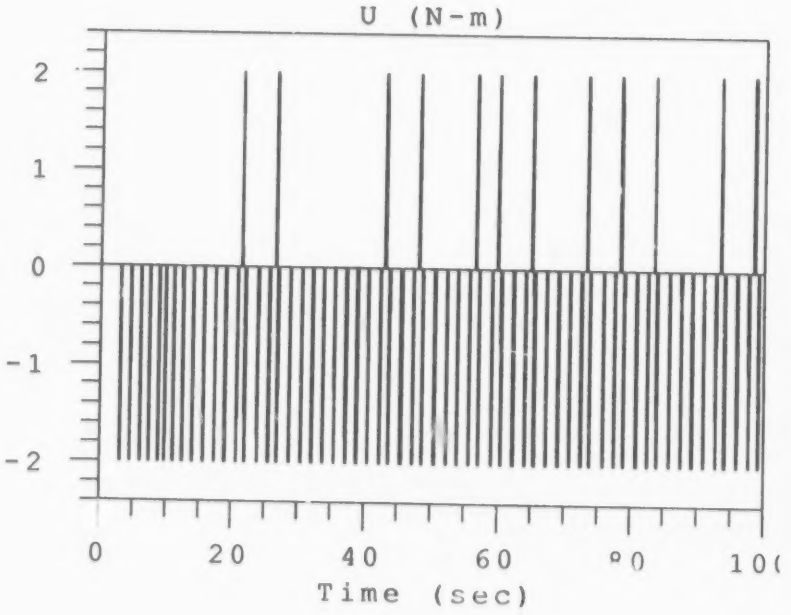
File: WORM.DAT

The time is 13:36:40.13 10-JUL-1988







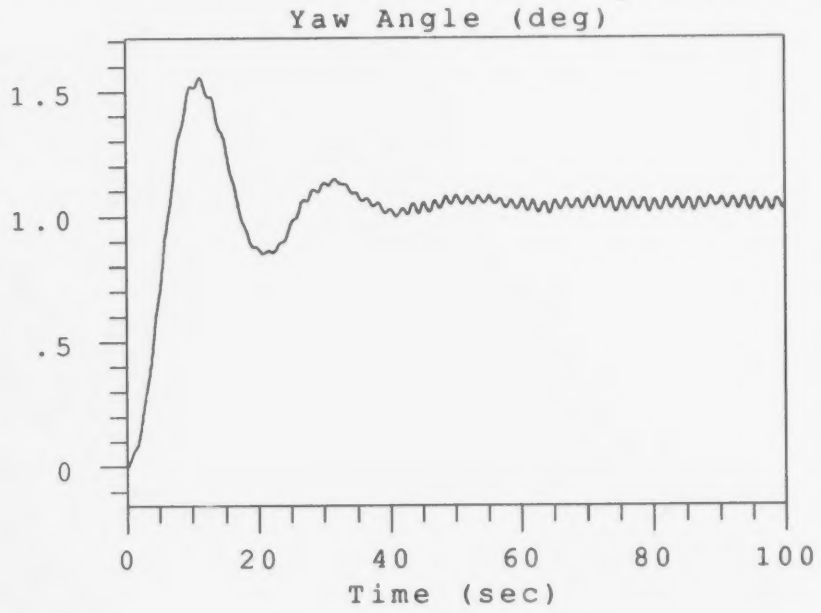


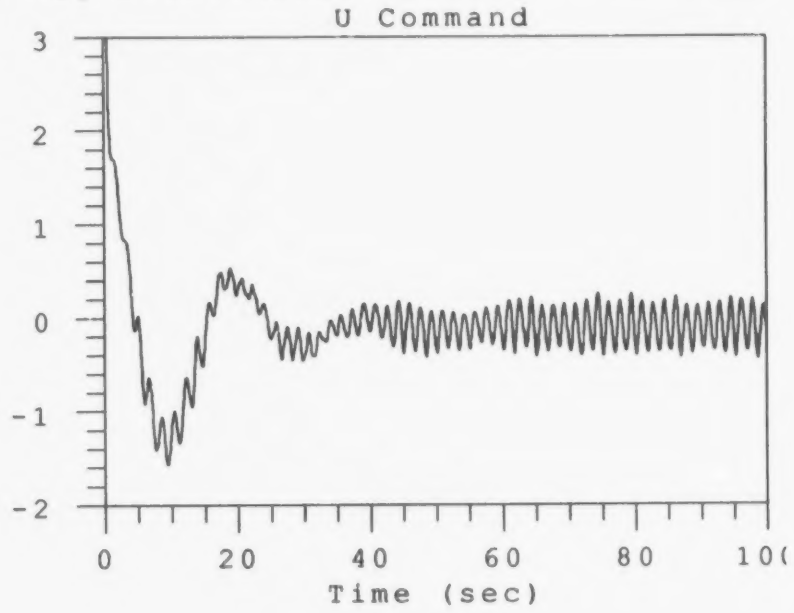
This system again represents case #1 except that torque and attitude commands are input to the system. As a result, the steady-state torque output looks much as the previous system did.

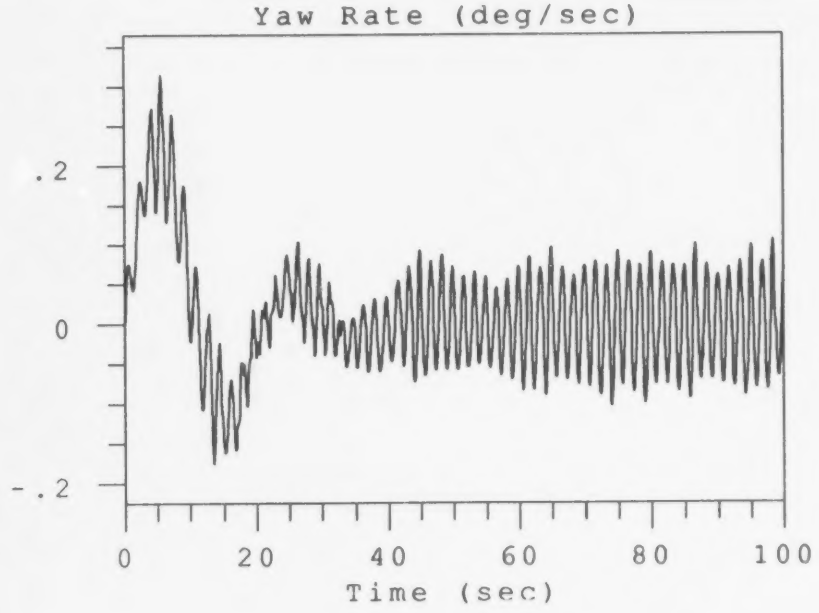
File: WORM.DAT

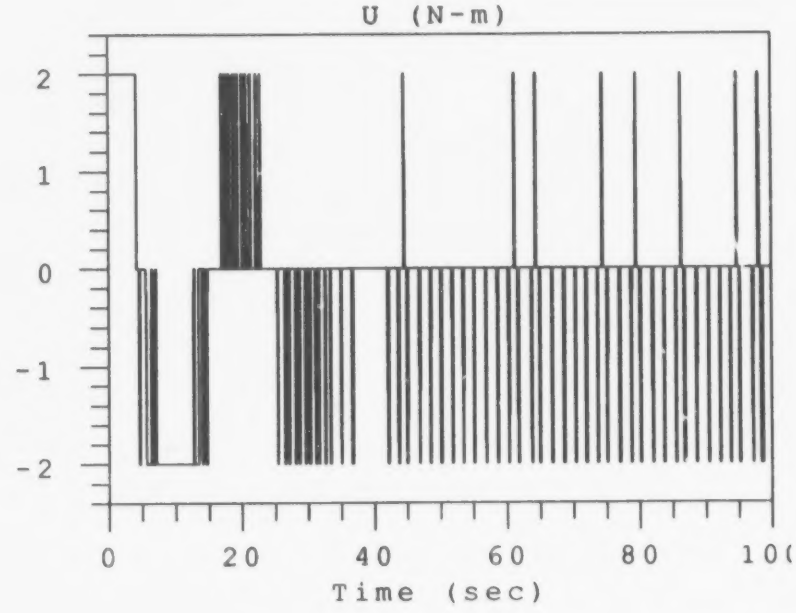
The time is 13:49:48.67 10-JUL-1988

Case #1 w/ delay & .2 N-m & 1 deg disturbance









Future INCA Enhancements*

- Incorporation of nonlinear simulation capability
- Incorporation of state-space control synthesis methodologies
- Incorporation of frequency-domain multivariable control synthesis methodologies

* efforts are within level of scope of present contract

Contents

- Introduction and project overview
  - Task descriptions
  - Project schedule
  - Tasks 1 & 2 accomplishments
- Introduction to INCA program
- Task 1: SIDF implementation
  - Nonlinear control elements
  - Single-input describing function (SIDF)
  - INCA enhancement for SIDF analysis
- Task 2: Nonlinear control synthesis using SIDF
- Plan for the second year: Tasks 3 & 4
- Summary and conclusions
- Appendix

Summary and Conclusions

- SIDF greatly reduces the amount of simulation time required for nonlinear control design.
- SIDF enables approximation of nonlinear gain and phase margins which can be verified by numerical simulation.
- There exists a need for a nonlinear simulation capability within INCA.

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- Introduction and project overview
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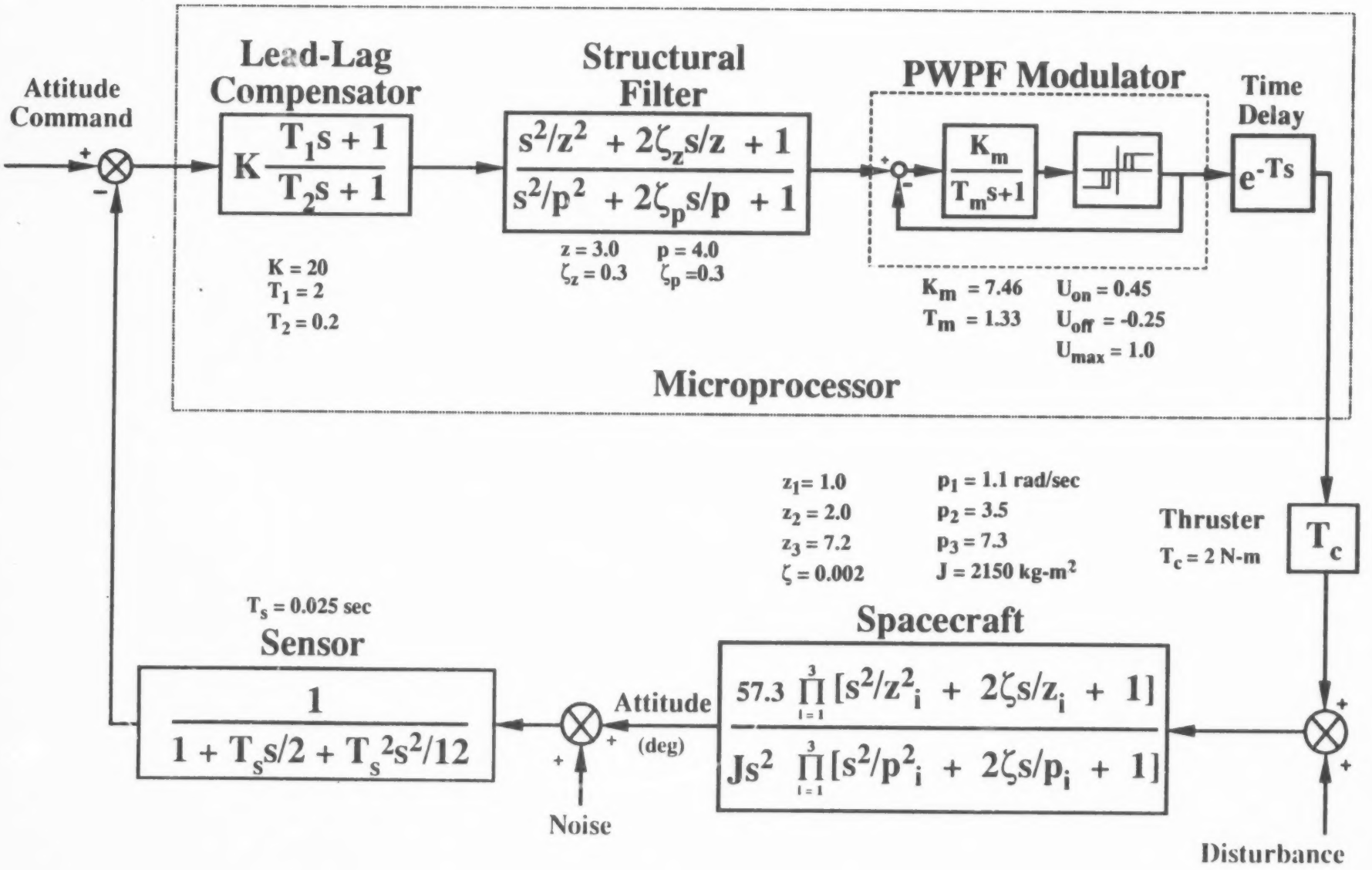
Appendix

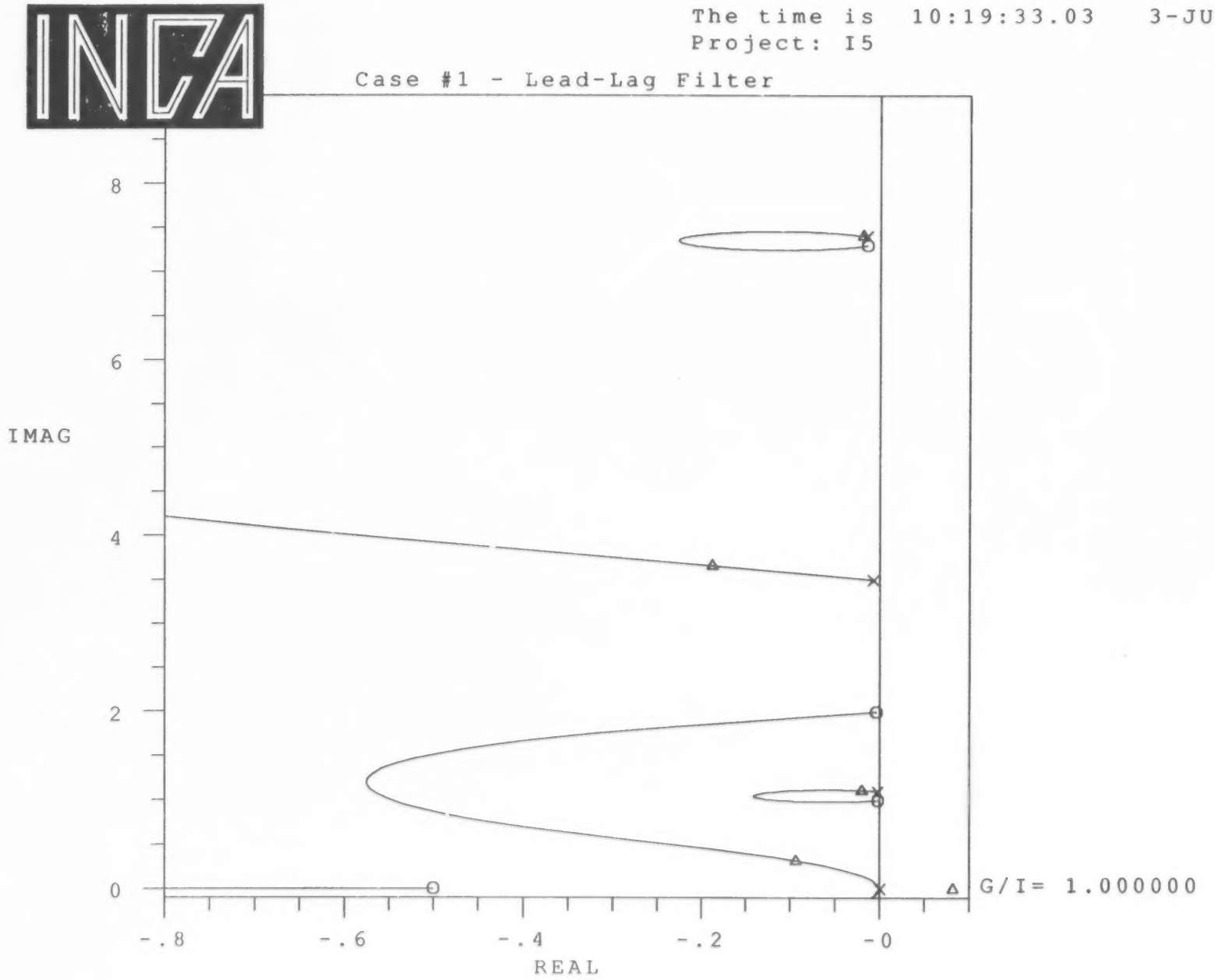
Linear Control Analysis of INTELSAT-5 Spacecraft Yaw-Axis RCS

Cases #1-5

Pulse-Modulated On-Off Control System for a Flexible Spacecraft

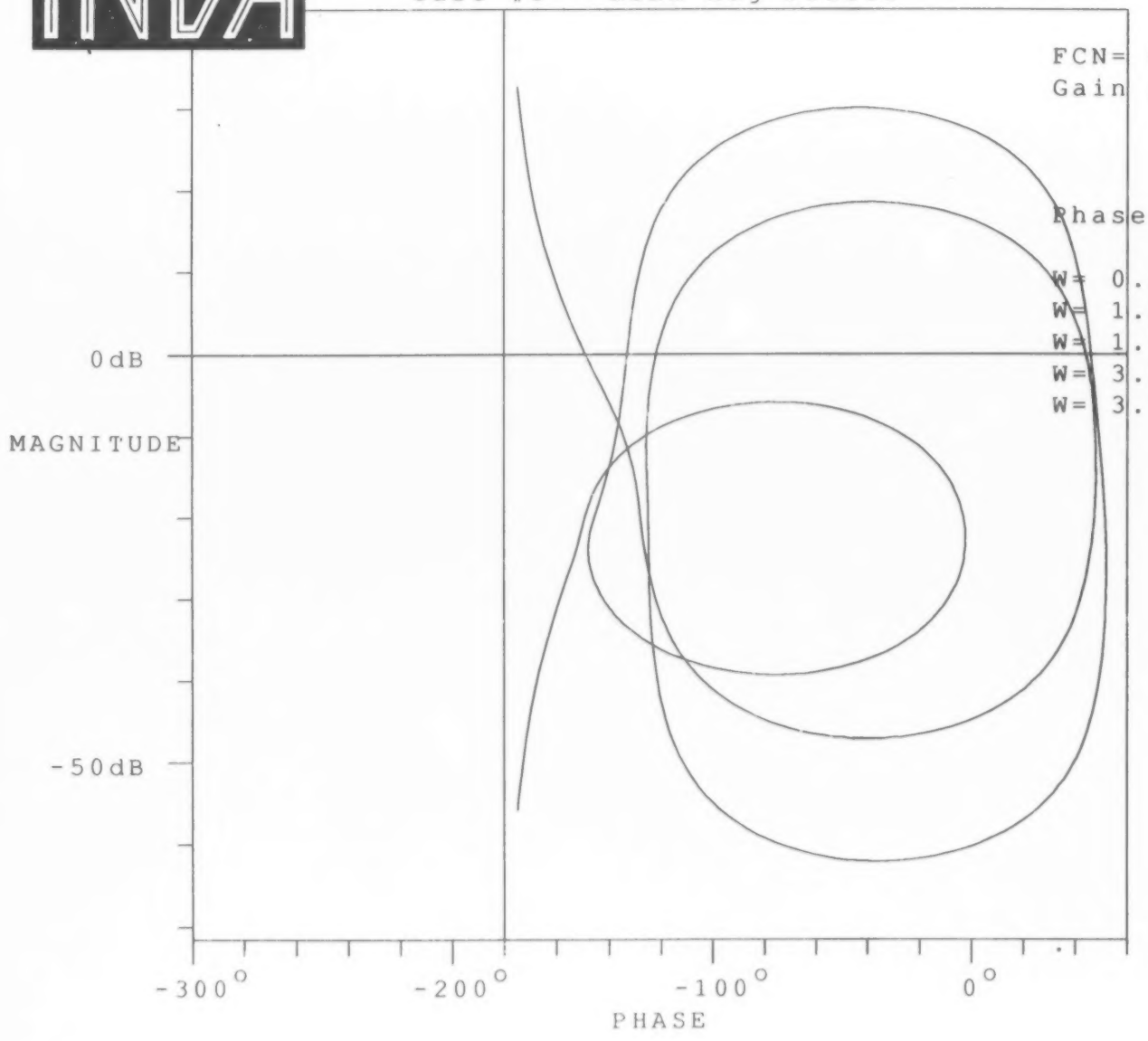

Attitude Command
Lead-Lag Compensator
$K\frac{T_1 s + 1}{T_2 s + 1}$
K = 20
T_1 = 2
T_2 = 0.2
Structural Filter
$\frac{s^2/z^2 + 2\zeta_z s/z + 1}{s^2/p^2 + 2\zeta_p s/p + 1}$
z = 3.0 p = 4.0
ζ_z = 0.3 ζ_p = 0.3
PWPF Modulator
$\frac{K_m}{T_m s+1}$
K_m = 7.46 U_{on} = 0.45
T_m = 1.33 U_{off} = -0.25
U_{max} = 1.0
Time Delay
e^{-Ts}
Microprocessor
z_1 = 1.0 p_1 = 1.1 rad/sec
z_2 = 2.0 p_2 = 3.5
z_3 = 7.2 p_3 = 7.3
ζ = 0.002 J = 2150 kg-m^2
Thruster
T_c = 2 N-m
T_c
T_s = 0.025 sec
Sensor
$\frac{1}{1 + T_s s/2 + T_s^2 s^2/12}$
Spacecraft
$\frac{57.3 \prod_{i=1}^{3} [s^2/z_i^2 + 2\zeta s/z_i + 1]}{Js^2 \prod_{i=1}^{3} [s^2/p_i^2 + 2\zeta s/p_i + 1]}$
Attitude (deg)
Noise
Disturbance

The time is 10:19:33.03 3-JUL-1988
Project: I5



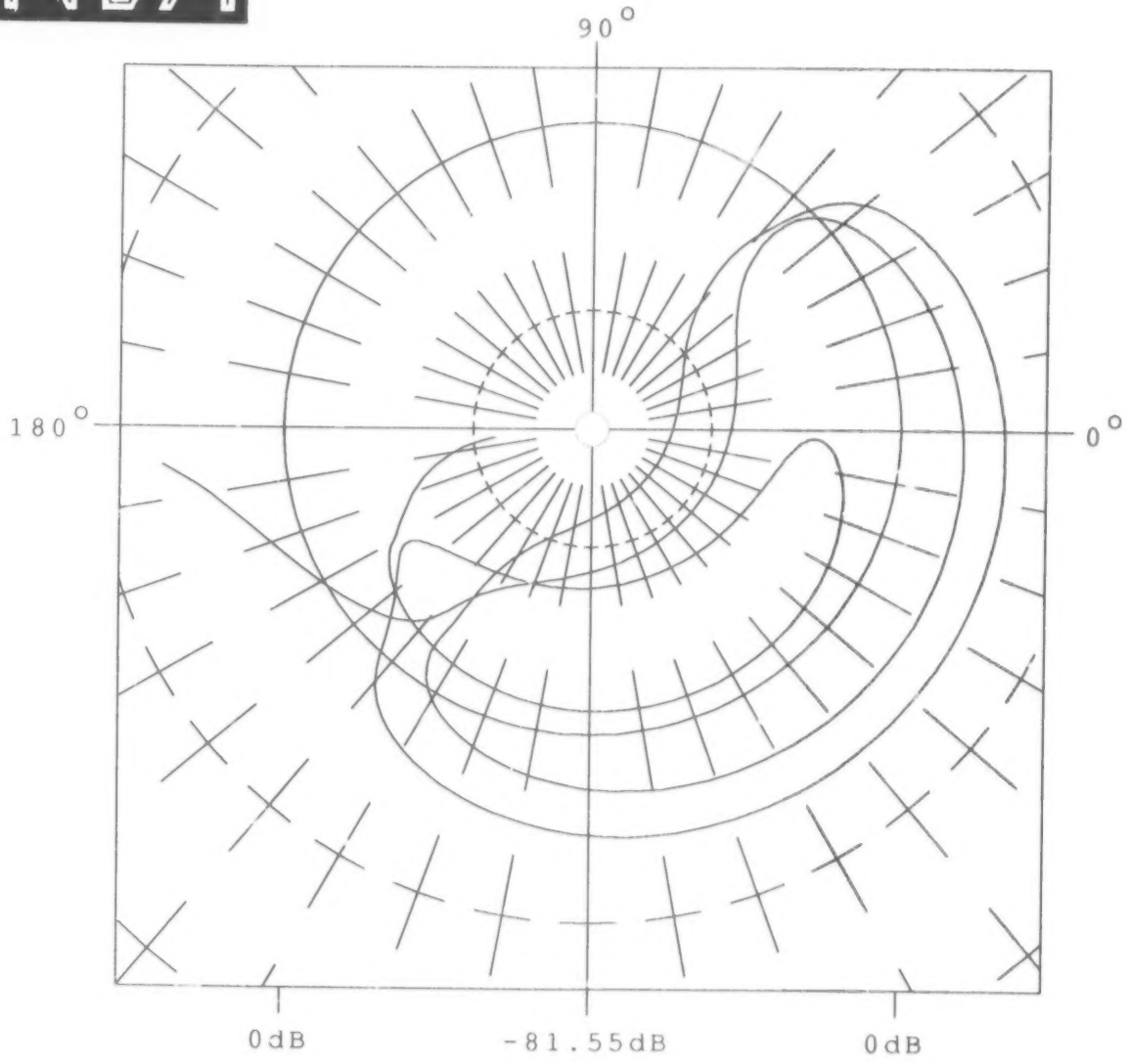
The time is 10:32:59.72 3-JUL-1988
Project: I5

Case #1 - Lead-Lag Filter

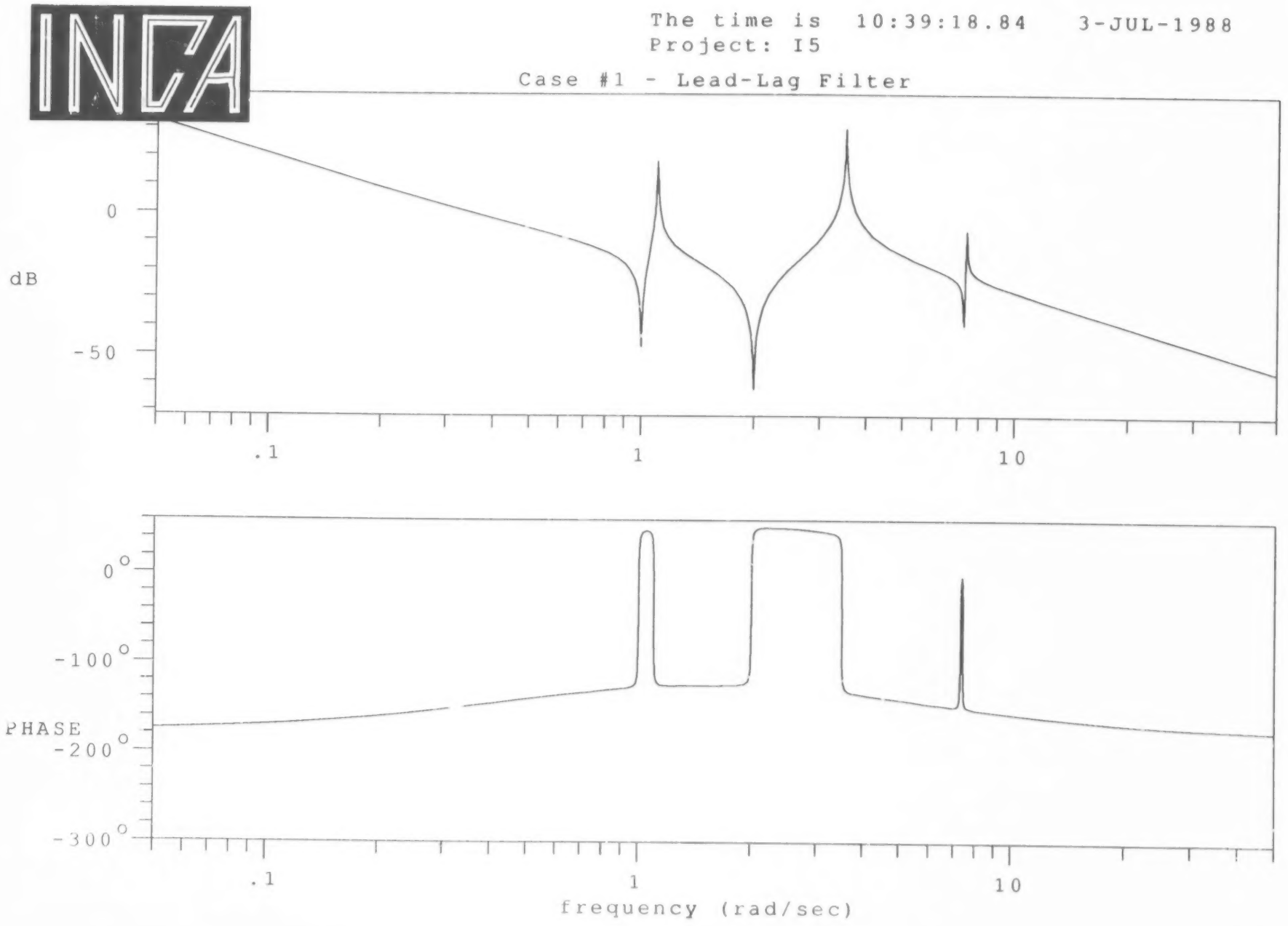


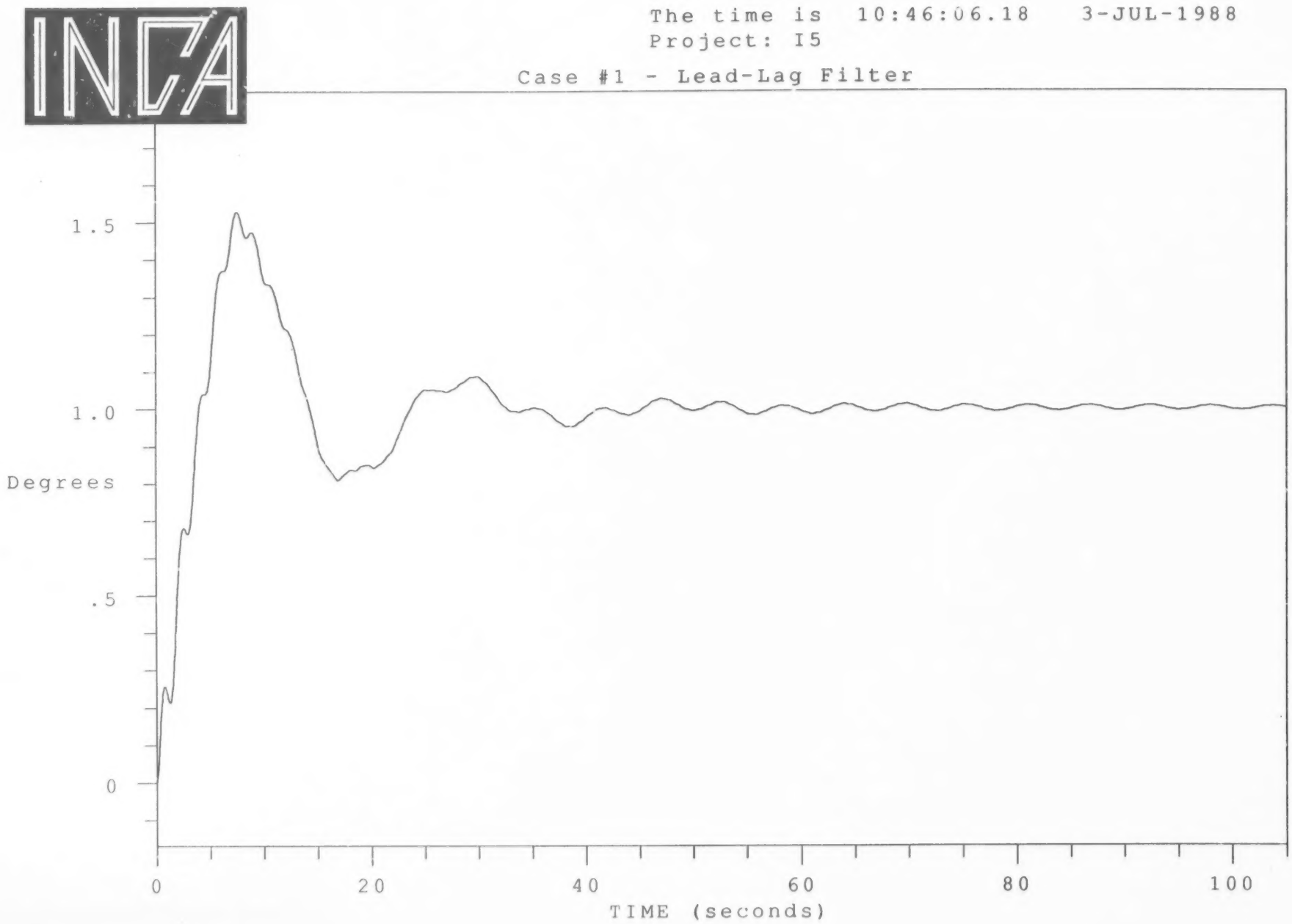
The time is 10:37:41.28 3-JUL-1988
Project: I5

Case #1 - Lead-Lag Filter

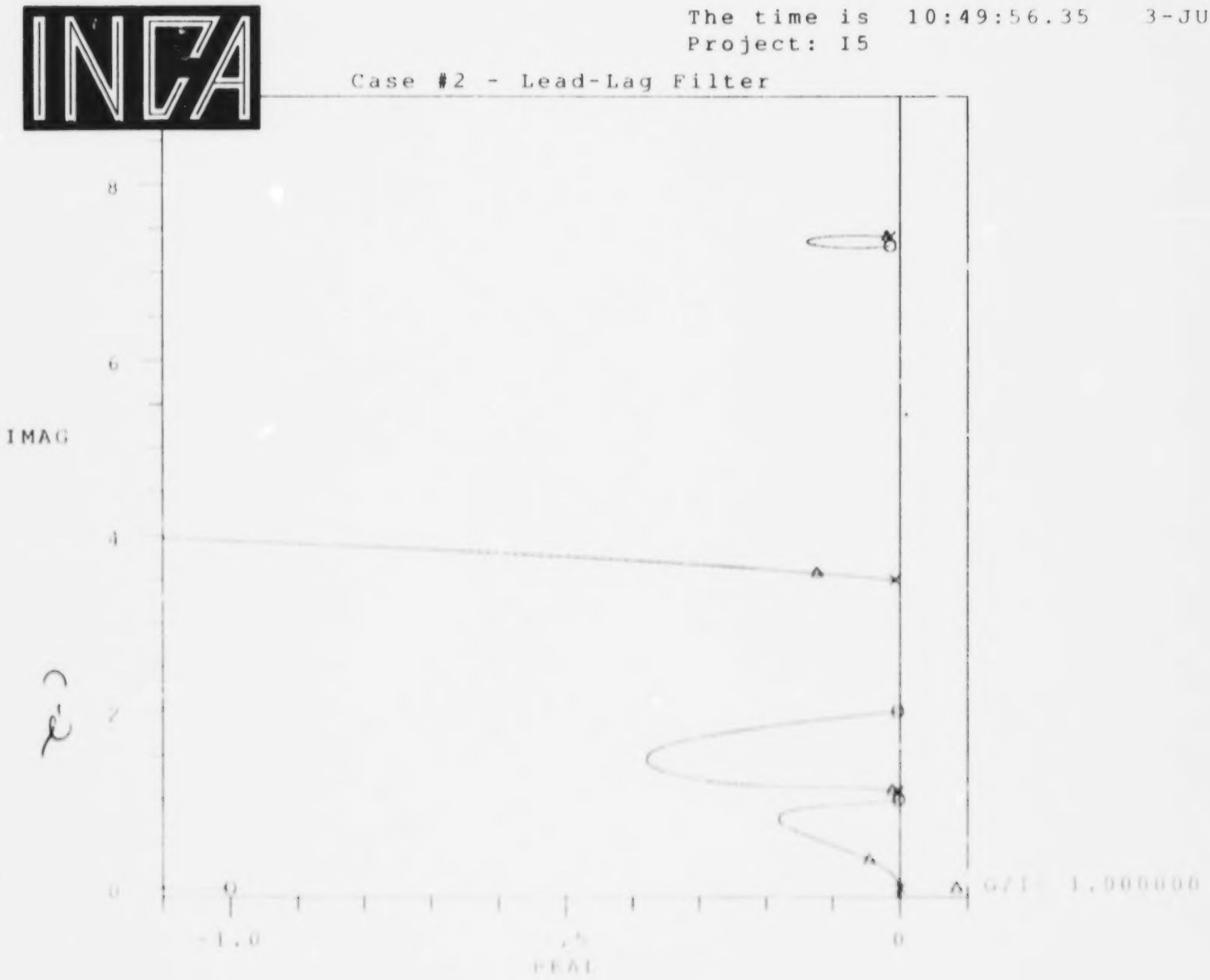


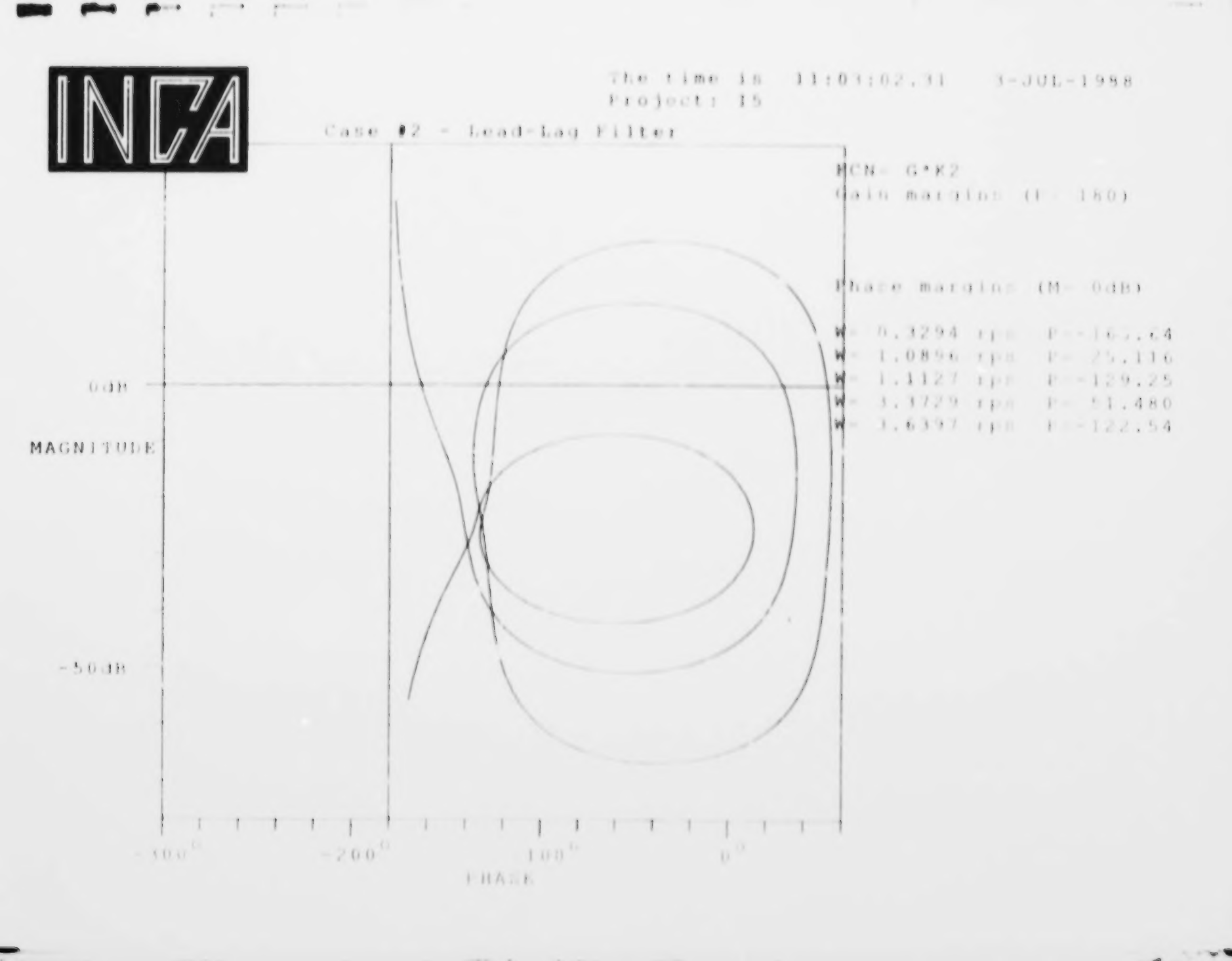
Origin=-81.55dB


INCA
The time is 10:39:18.84 3-JUL-1988
Project: I5
Case #1 - Lead-Lag Filter
0
dB
-50
.1
1
10
0°
-100°
PHASE
-200°
-300°
.1
1
10
frequency (rad/sec)

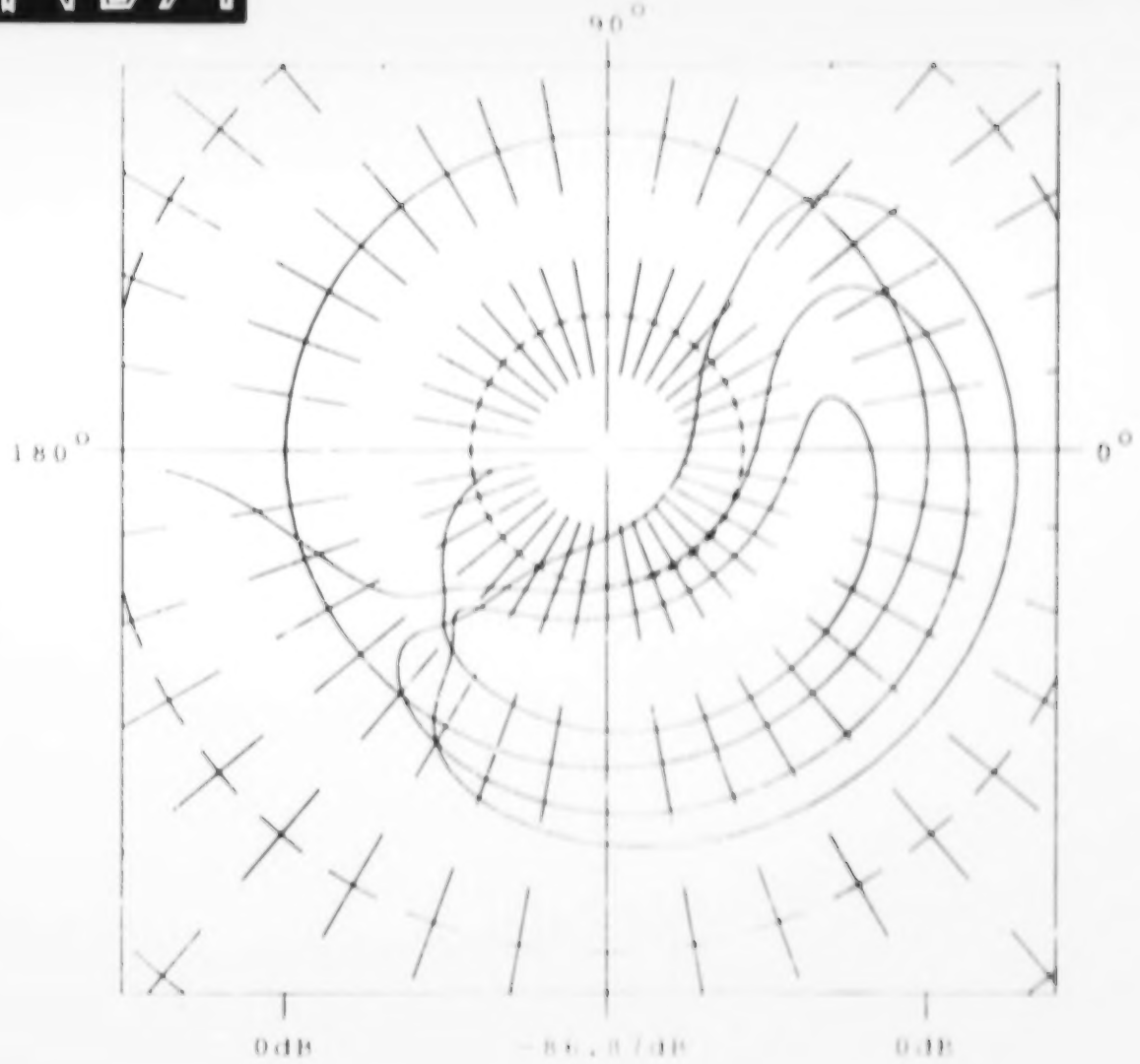

The time is 10:46:06.18 3-JUL-1988
Project: I5
Case #1 - Lead-Lag Filter
1.5
1.0
Degrees
.5
0
0
20
40
60
80
100
TIME (seconds)

The time is 10:49:56.35 3-JUL-1988
Project: I5

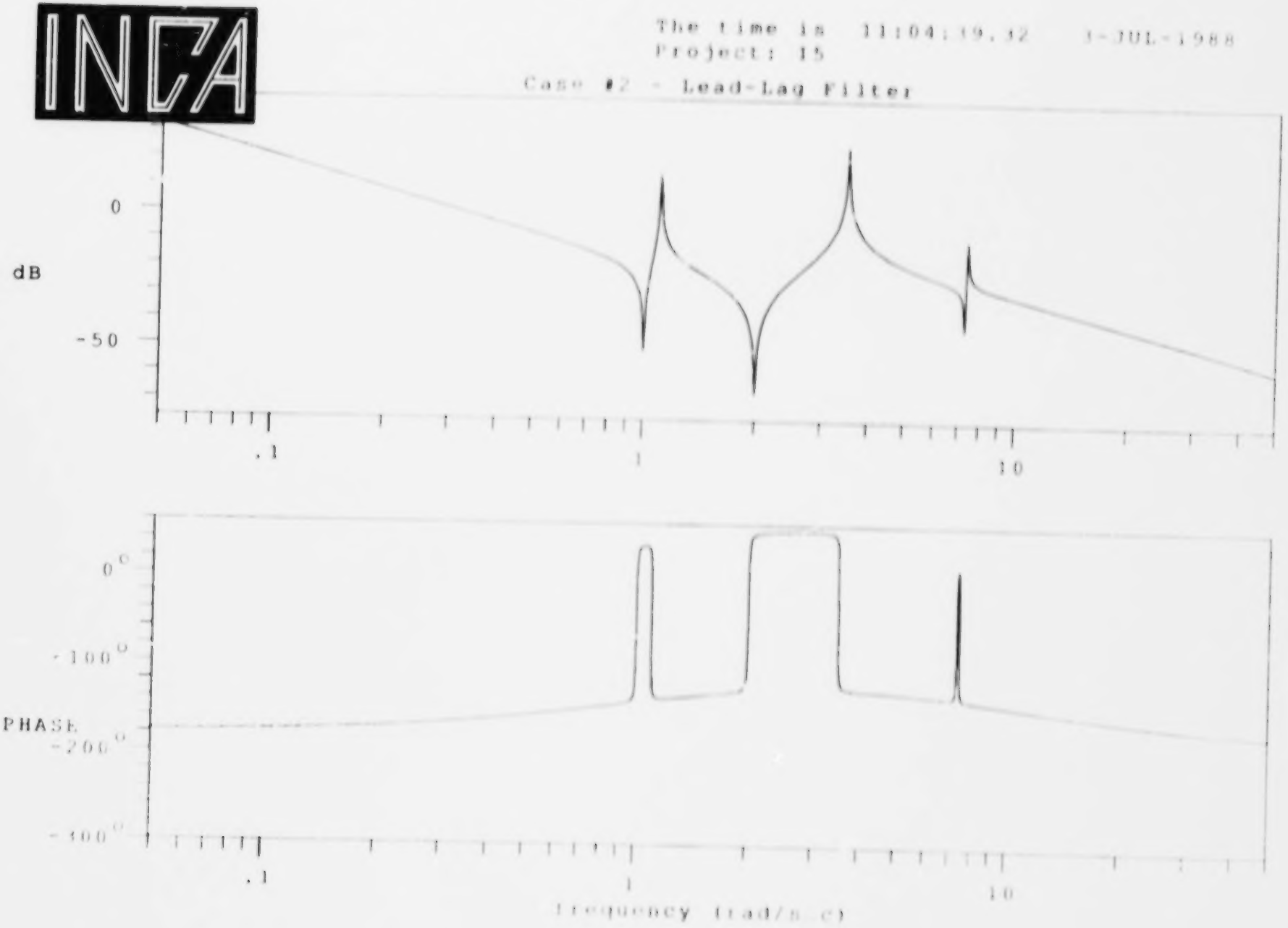



INCA
The time is 11:01:02.31 3-JUL-1988
Project: 15
Case #2 - Lead-Lag Filter
HCN= G*K2
Gain margin: (M= 16D)
Phase margin: (M= 0dB)
W= 0.3294 rps P= 164.64
W= 1.0896 rps P= 25.116
W= 1.1127 rps P=-129.25
W= 3.3729 rps P= 51.480
W= 3.6397 rps P=-122.54
0dB
MAGNITUDE
-50dB
-300°
-200°
100°
0°
PHASE

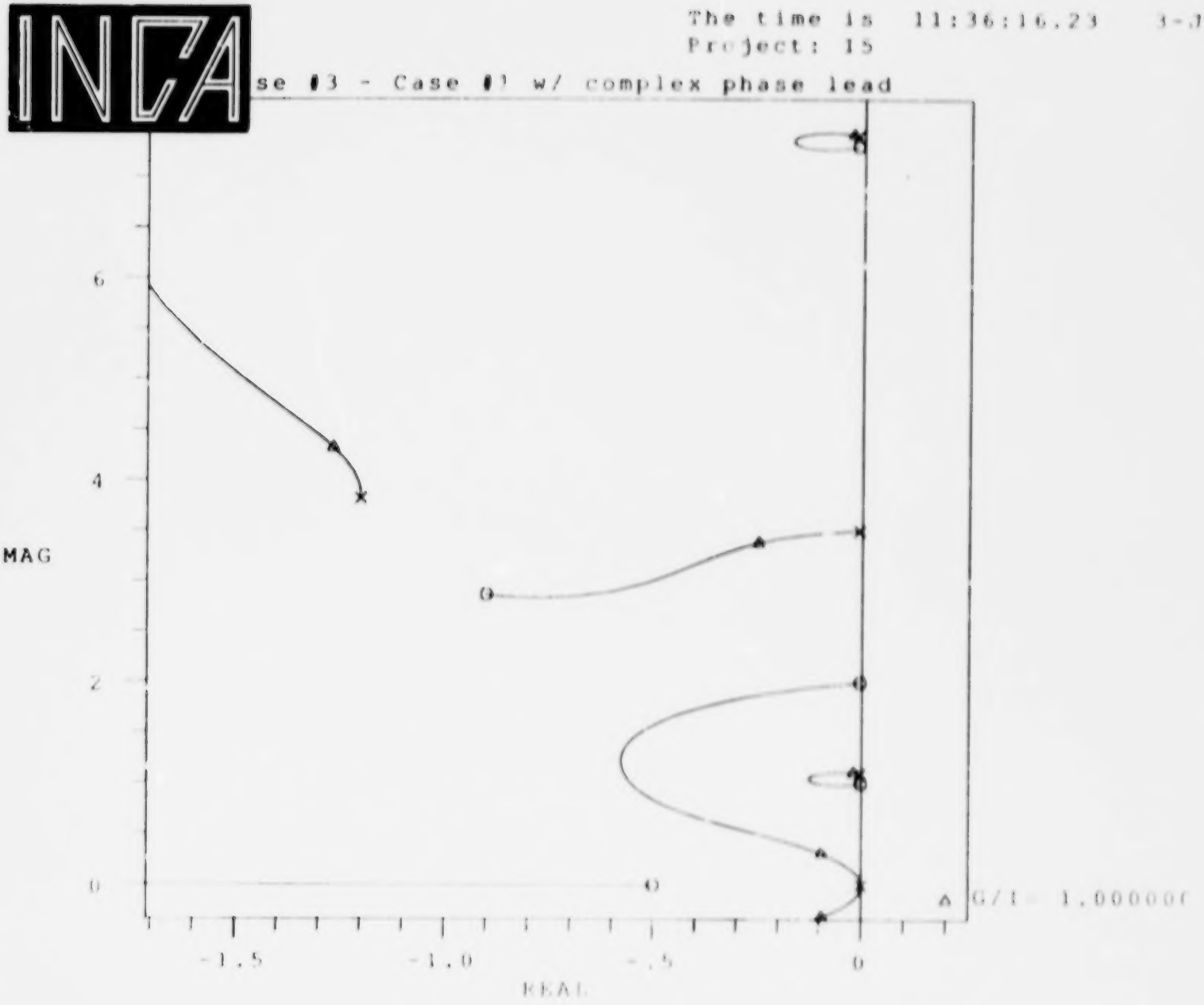
Case #2 - Lead-Lag Filter

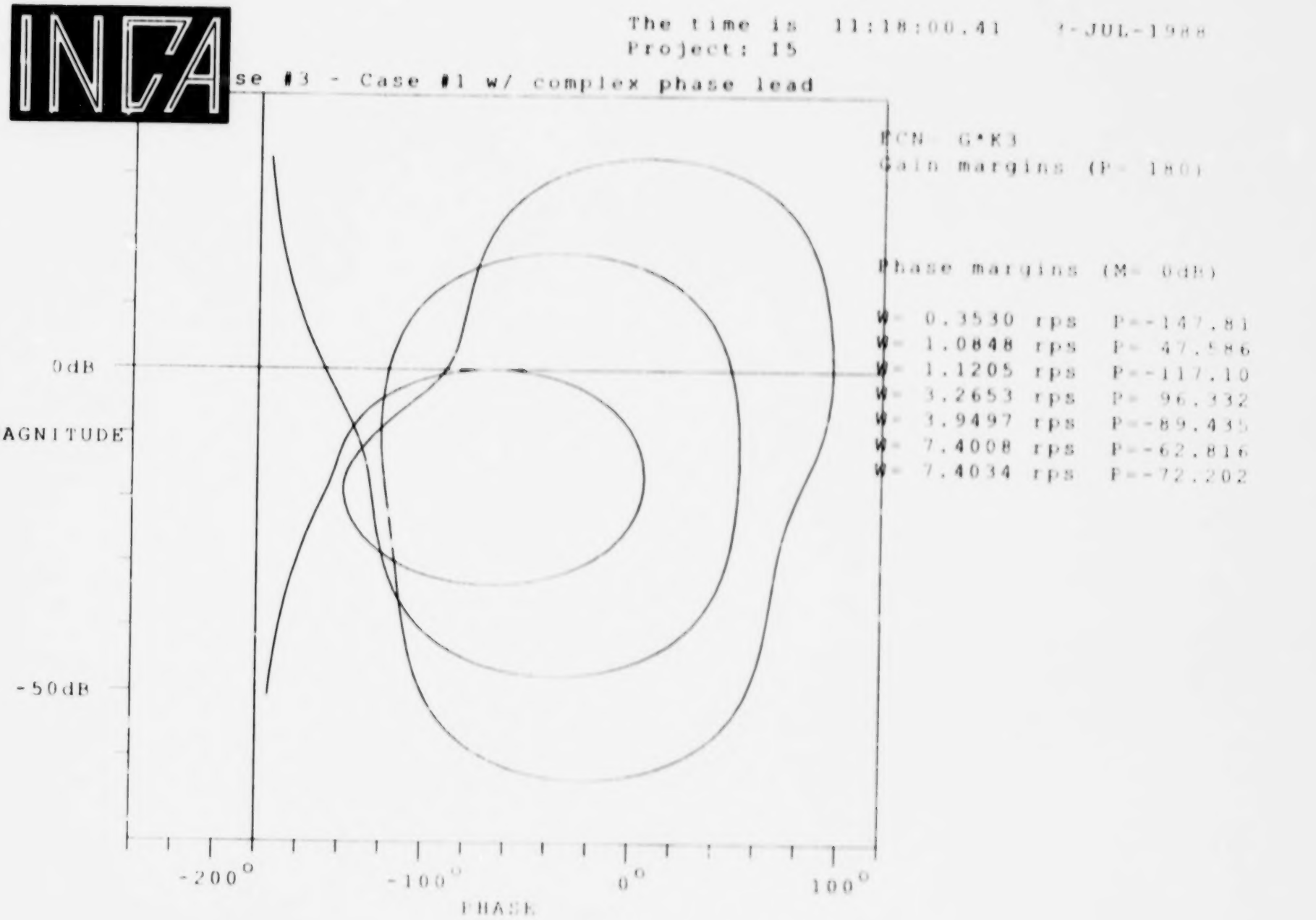


Origin= -86.87dB


INCA
The time is 11:04:19.32 1-JUL-1988
Project: 15
Case #2 - Lead-Lag Filter
0
dB
-50
.1
1
10
0°
-100°
PHASE
-200°
-300°
.1
1
10
Frequency (rad/s ec)

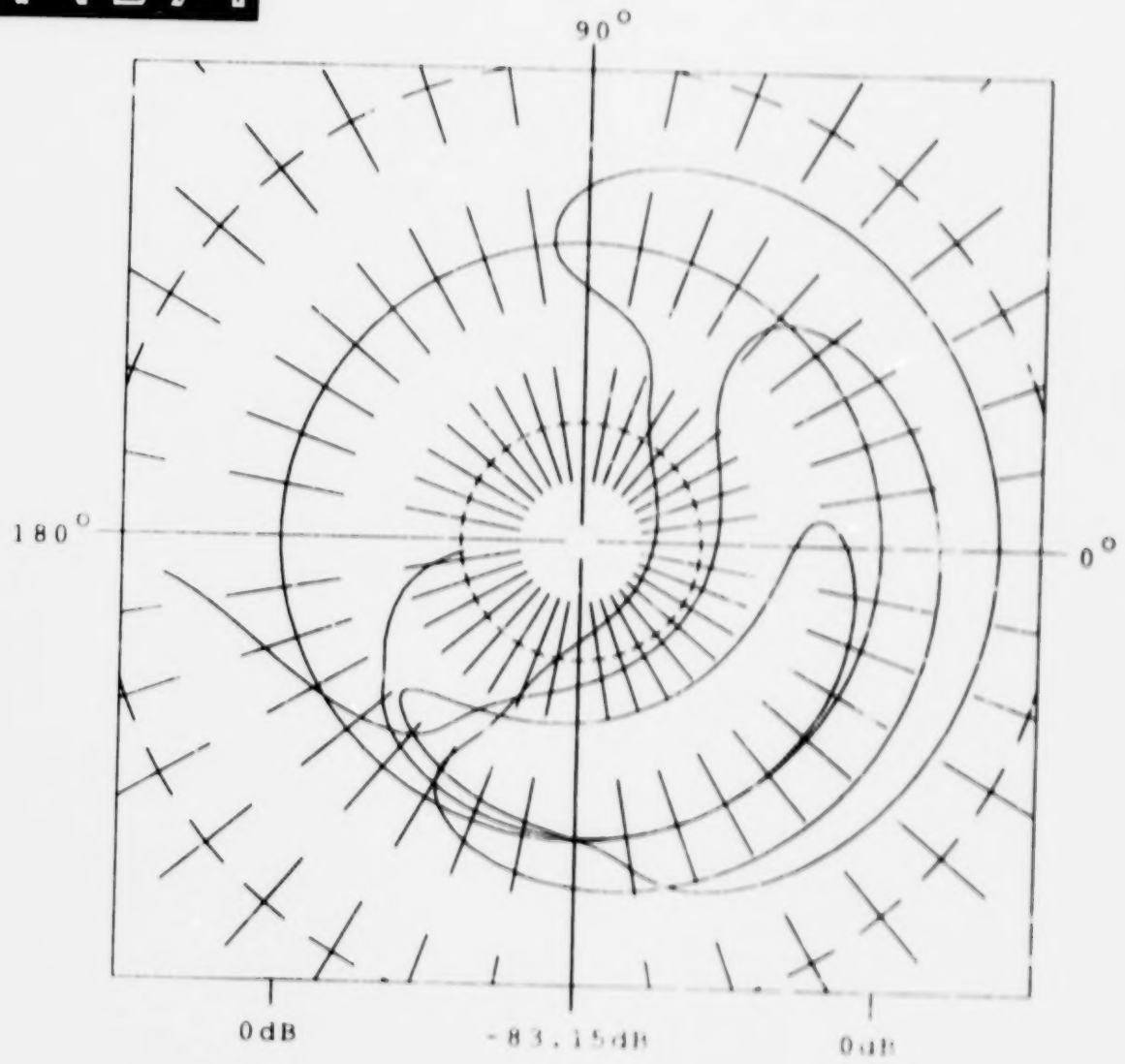
INCA
The time is 10:55:08.31 3-JUL-1988
Project: 15
Case #2 - Lead-Lag Filter
1.5
1.0
Degrees
.5
0
0
20
40
60
80
100
TIME (seconds)


The time is 11:36:16.23 3-JUL-1988
Project: 15
se #3 - Case #1 w/ complex phase lead
6
4
IMAG
2
0
G/I = 1.000000
-1.5
-1.0
-.5
0
REAL

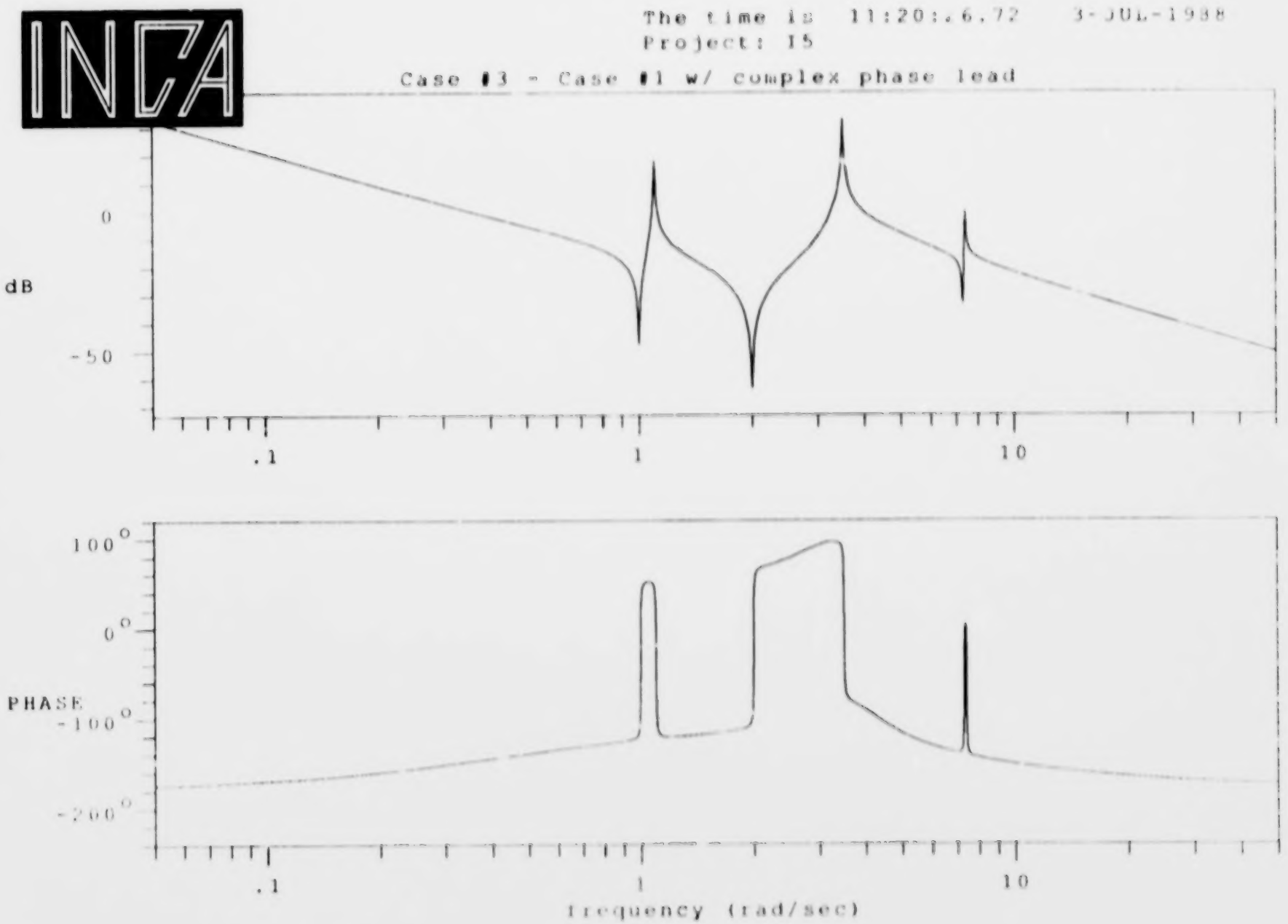

INCA
The time is 11:18:00.41 8-JUL-1988
Project: 15
se #3 - Case #1 w/ complex phase lead
FCN= G*K3
Gain margins (P= 180)
Phase margins (M= 0dB)
W= 0.3530 rps P=-147.81
W= 1.0848 rps P= 47.586
W= 1.1205 rps P=-117.10
W= 3.2653 rps P= 96.332
W= 3.9497 rps P=-89.435
W= 7.4008 rps P=-62.816
W= 7.4034 rps P=-72.202
0dB
MAGNITUDE
-50dB
-200°
-100°
0°
100°
PHASE

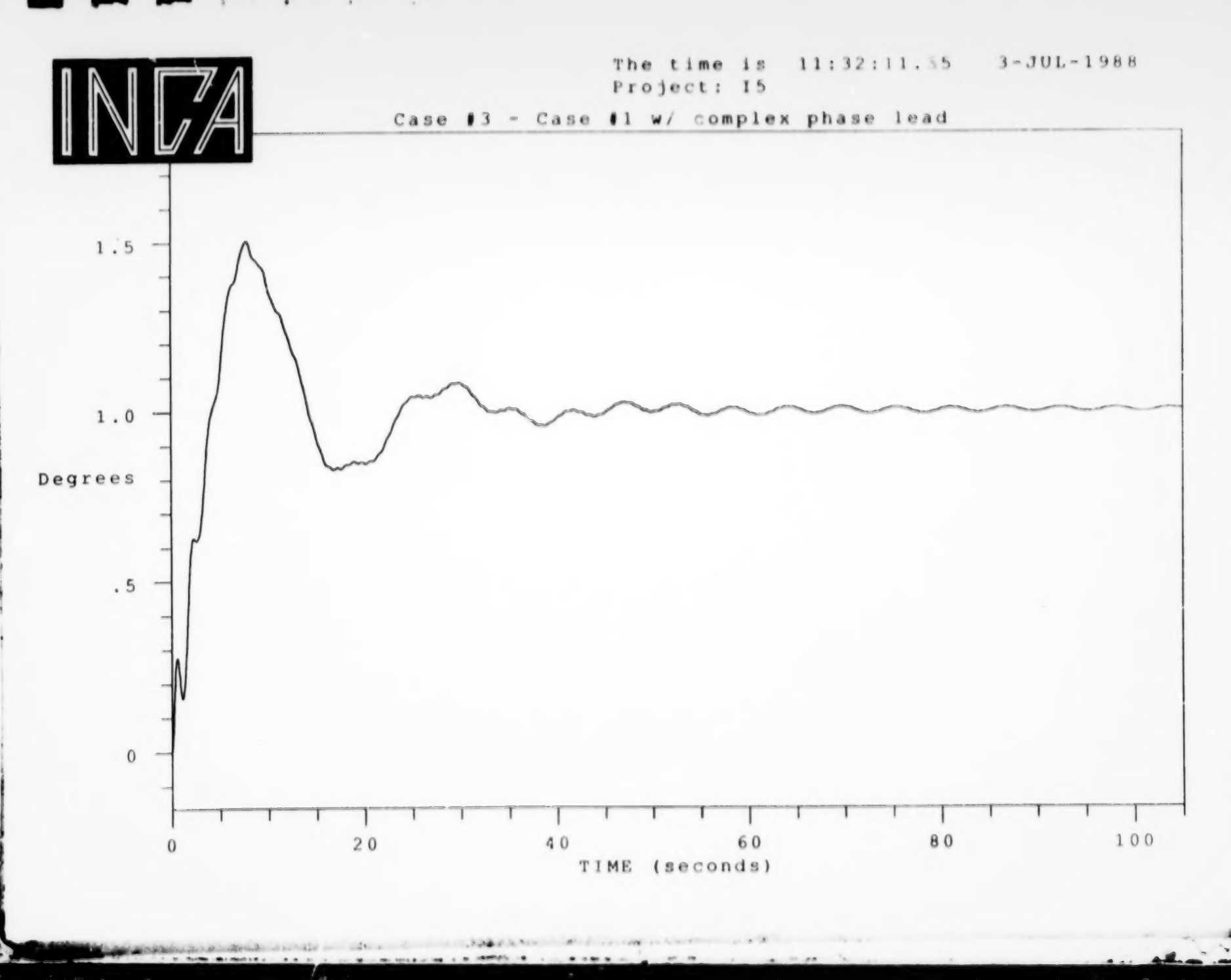
INCA

#3 - Case #1 w/ complex phase lead

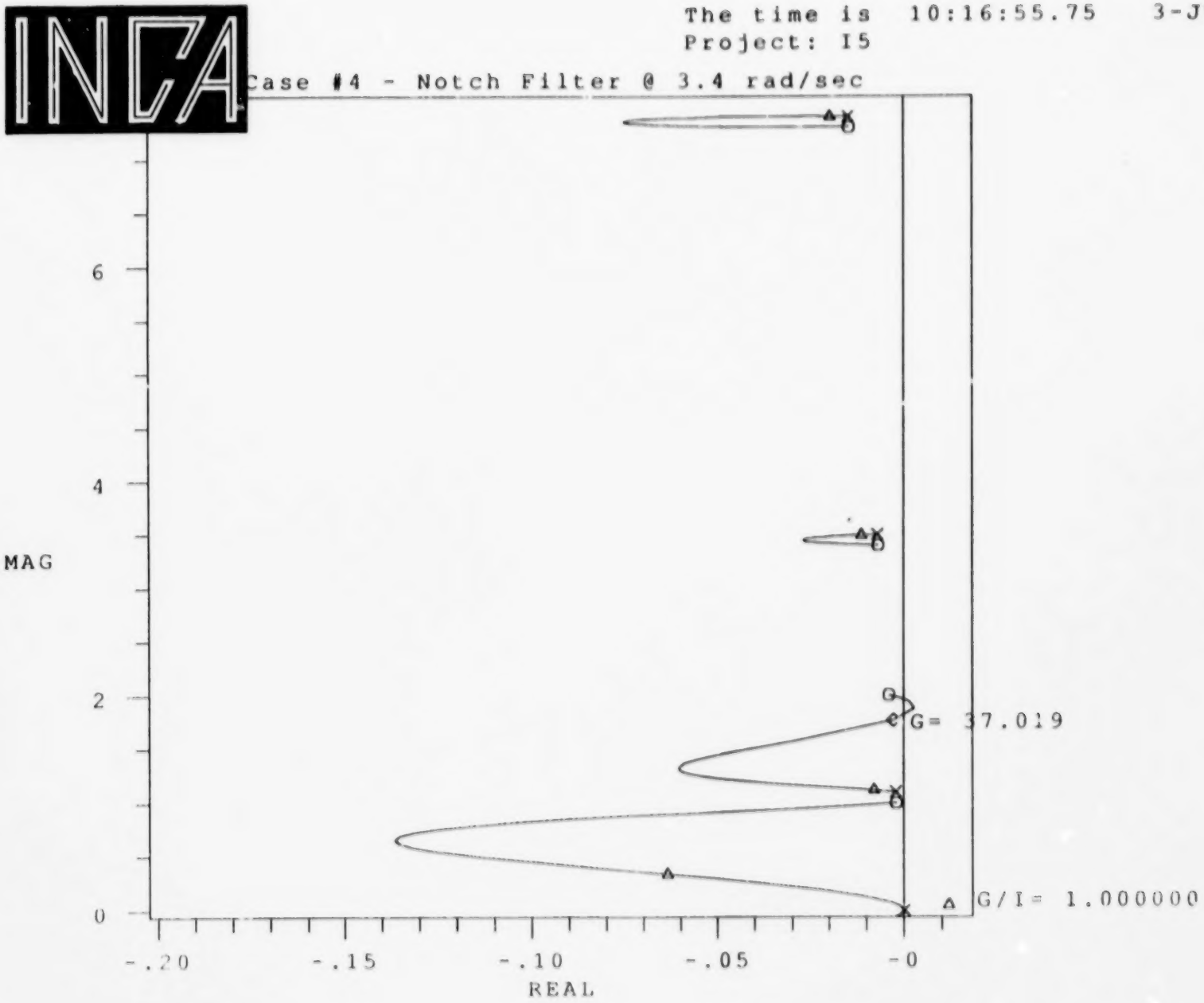


Origin=-83.15dB


The time is 11:20:.6.72 3-JUL-1988
Project: I5
Case #3 - Case #1 w/ complex phase lead
0
dB
-50
.1
1
10
100°
0°
PHASE
-100°
-200°
.1
1
10
frequency (rad/sec)

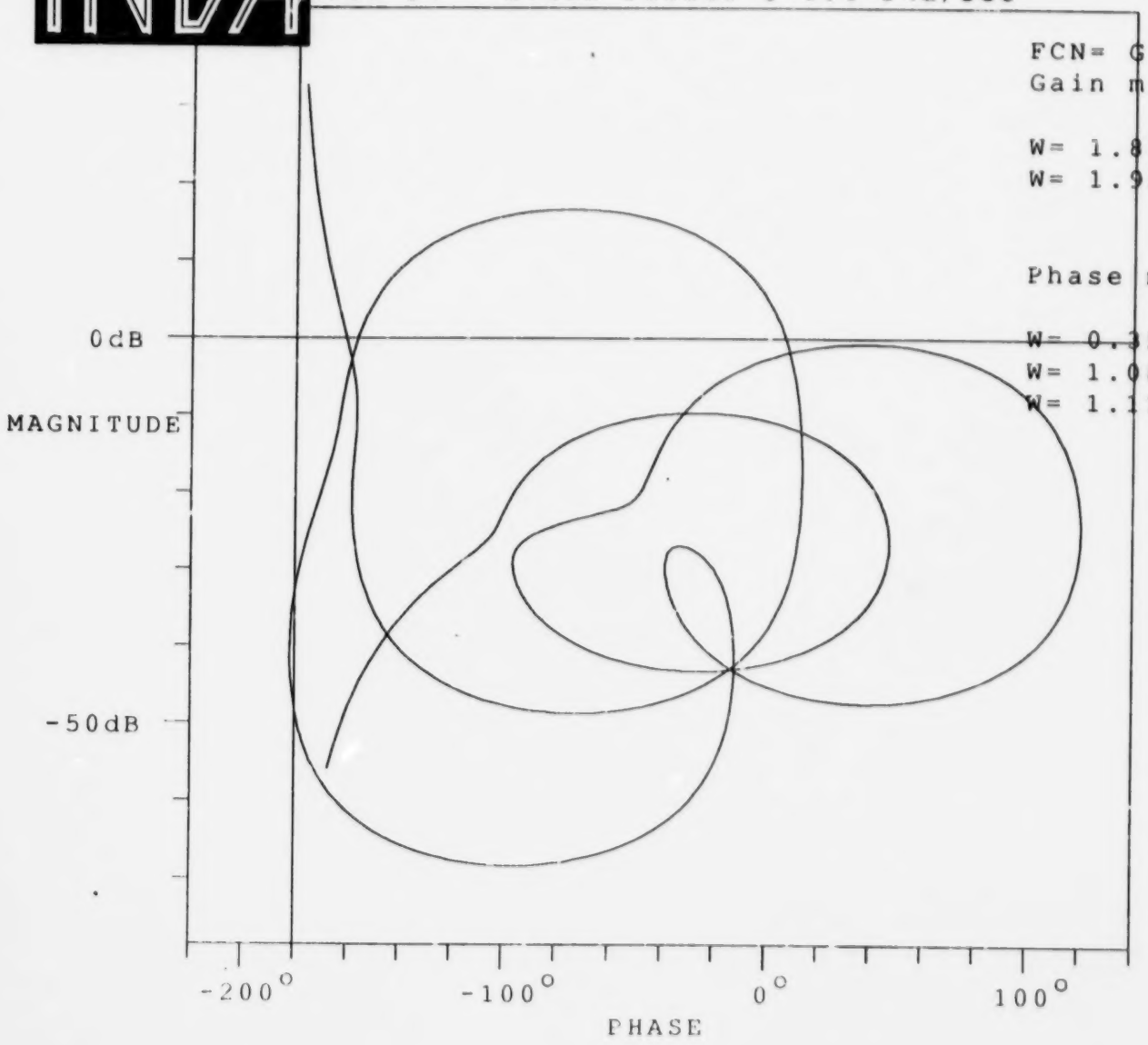

INCA
The time is 11:32:11.55 3-JUL-1988
Project: I5
Case #3 - Case #1 w/ complex phase lead
1.5
1.0
Degrees
.5
0
0
20
40
60
80
100
TIME (seconds)

The time is 10:16:55.75 3-JUL-1988
Project: I5



The time is 11:47:06.88 3-JUL-1988
Project: I5

Case #4 - Notch Filter @ 3.4 rad/sec



FCN= G*K4
Gain margins (P= 180)

W= 1.8022 rps M=-33.224 dB
W= 1.9594 rps M=-47.921 dB

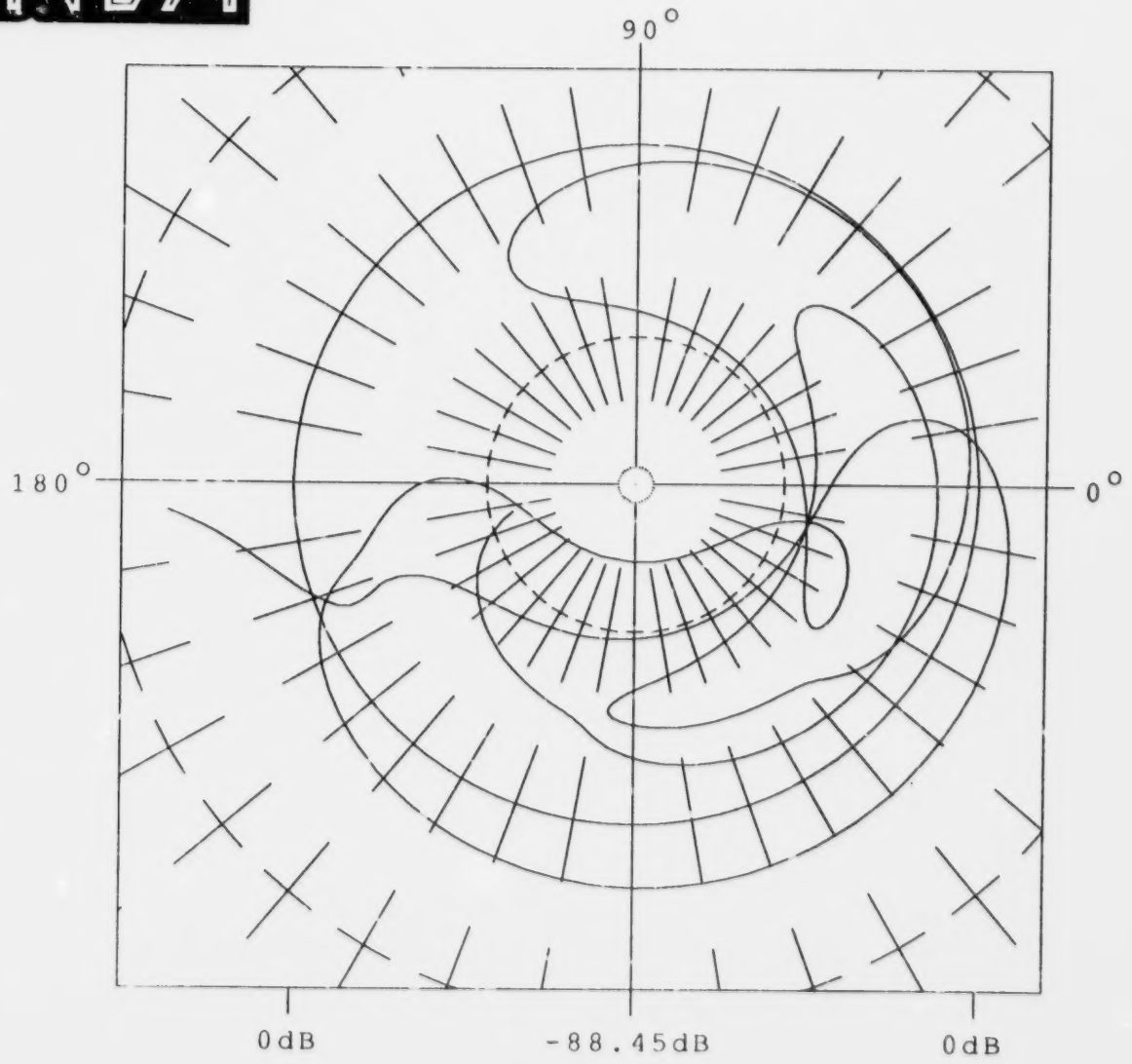
Phase margins (M= 0dB)

W= 0.3522 rps P=-160.83
W= 1.0868 rps P= 7.0080
W= 1.1170 rps P=-156.38

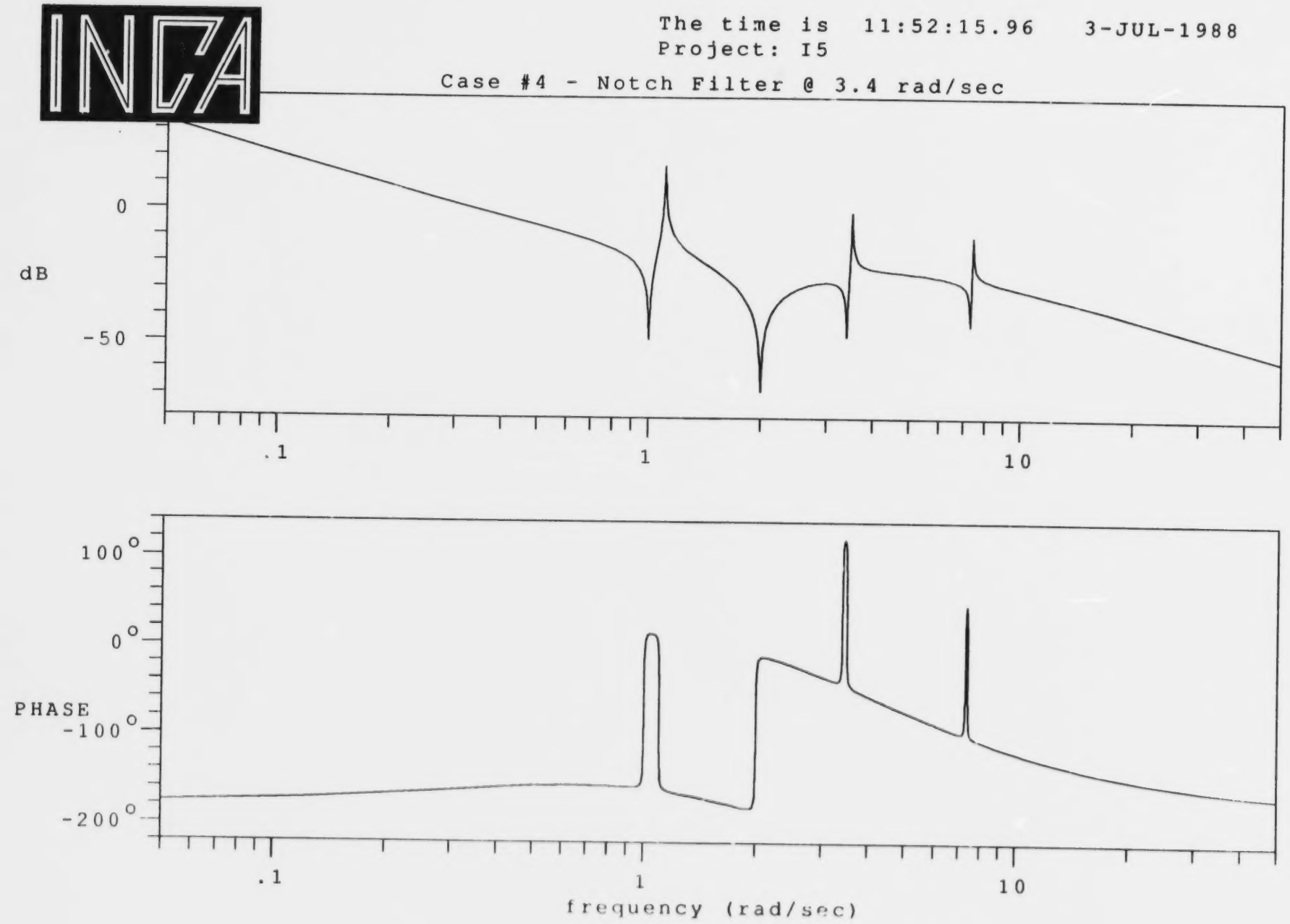
The time is 11:49:16.95 3-JUL-1988

Project: I5

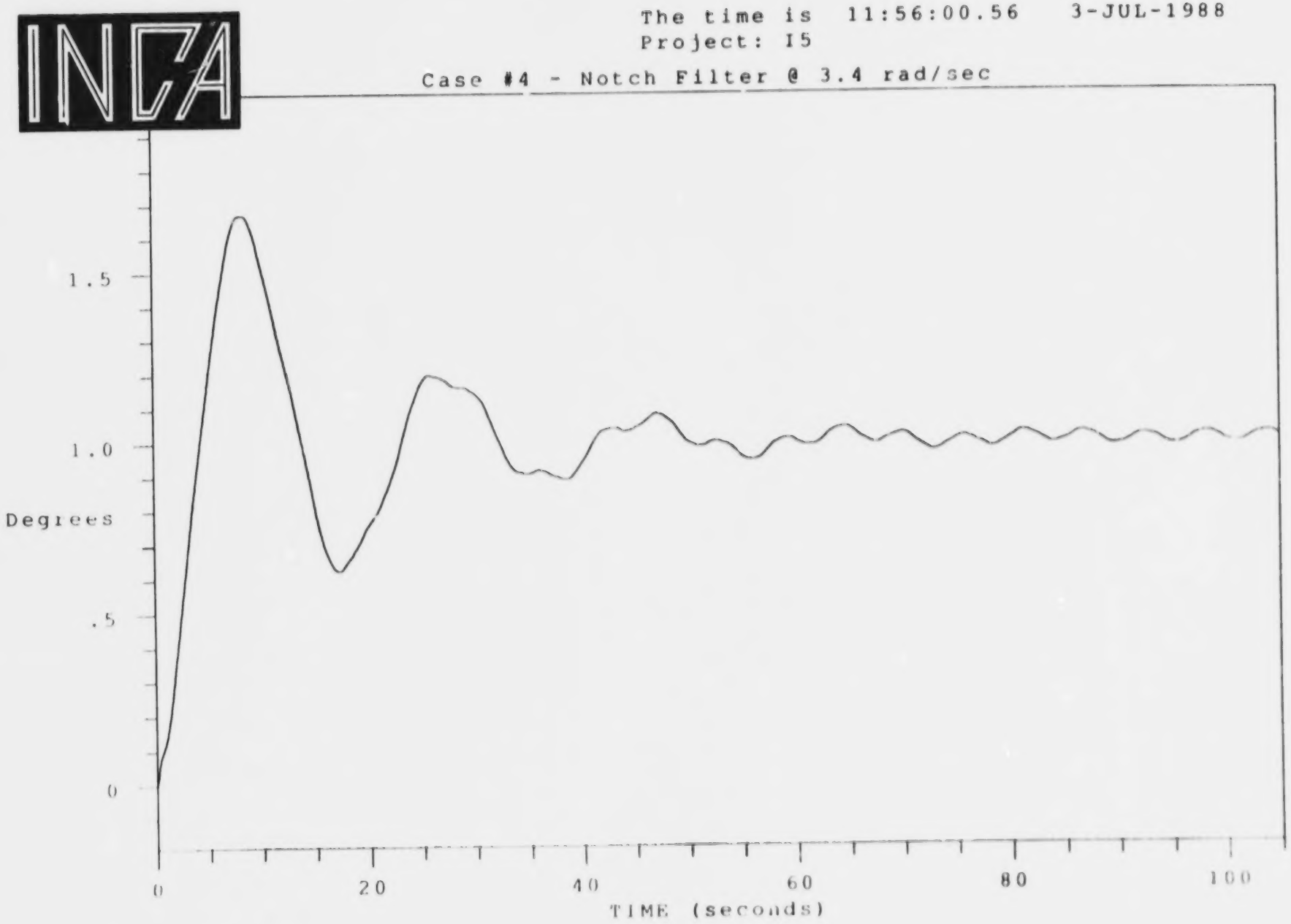
se #4 - Notch Filter @ 3.4 rad/sec

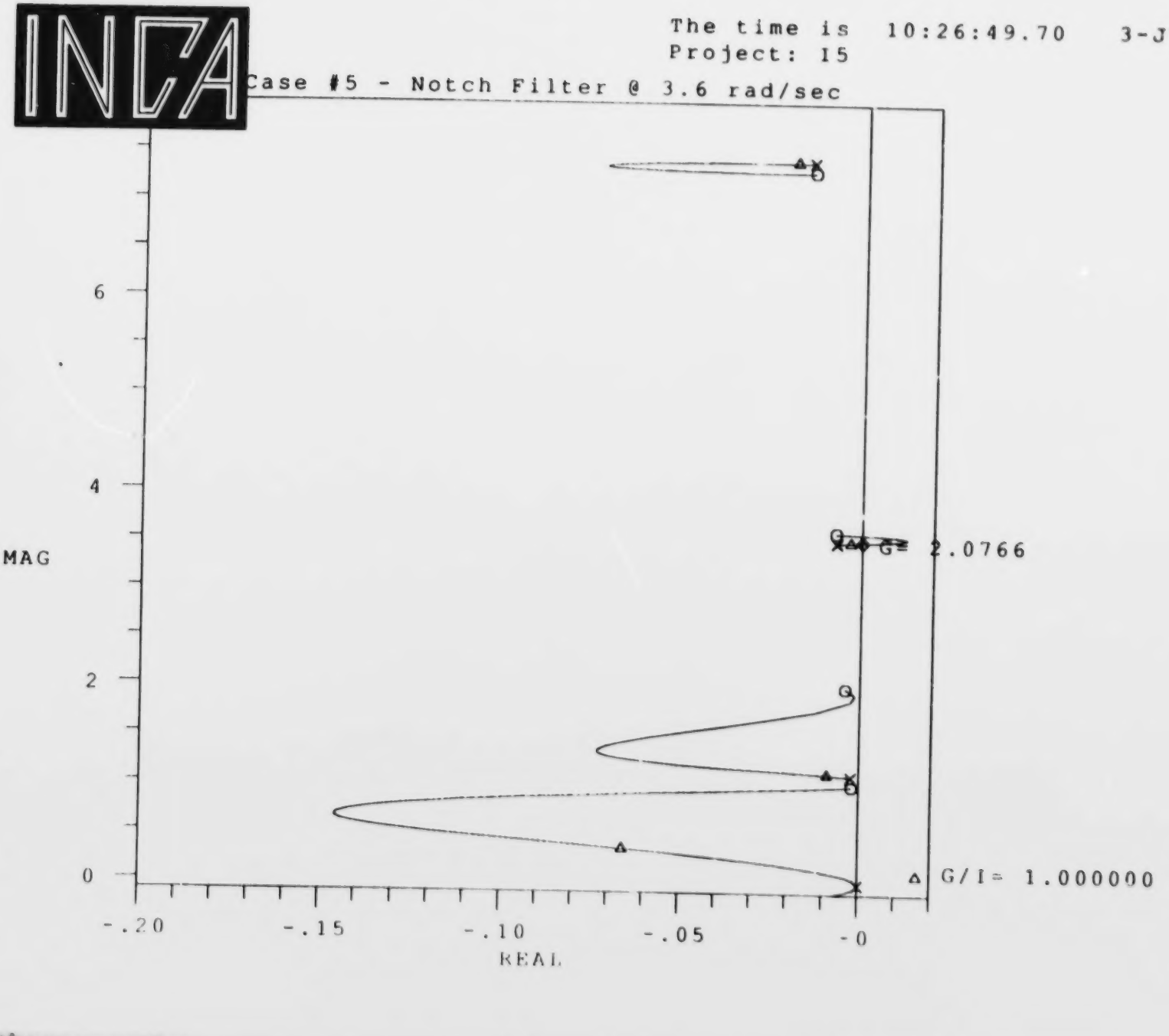


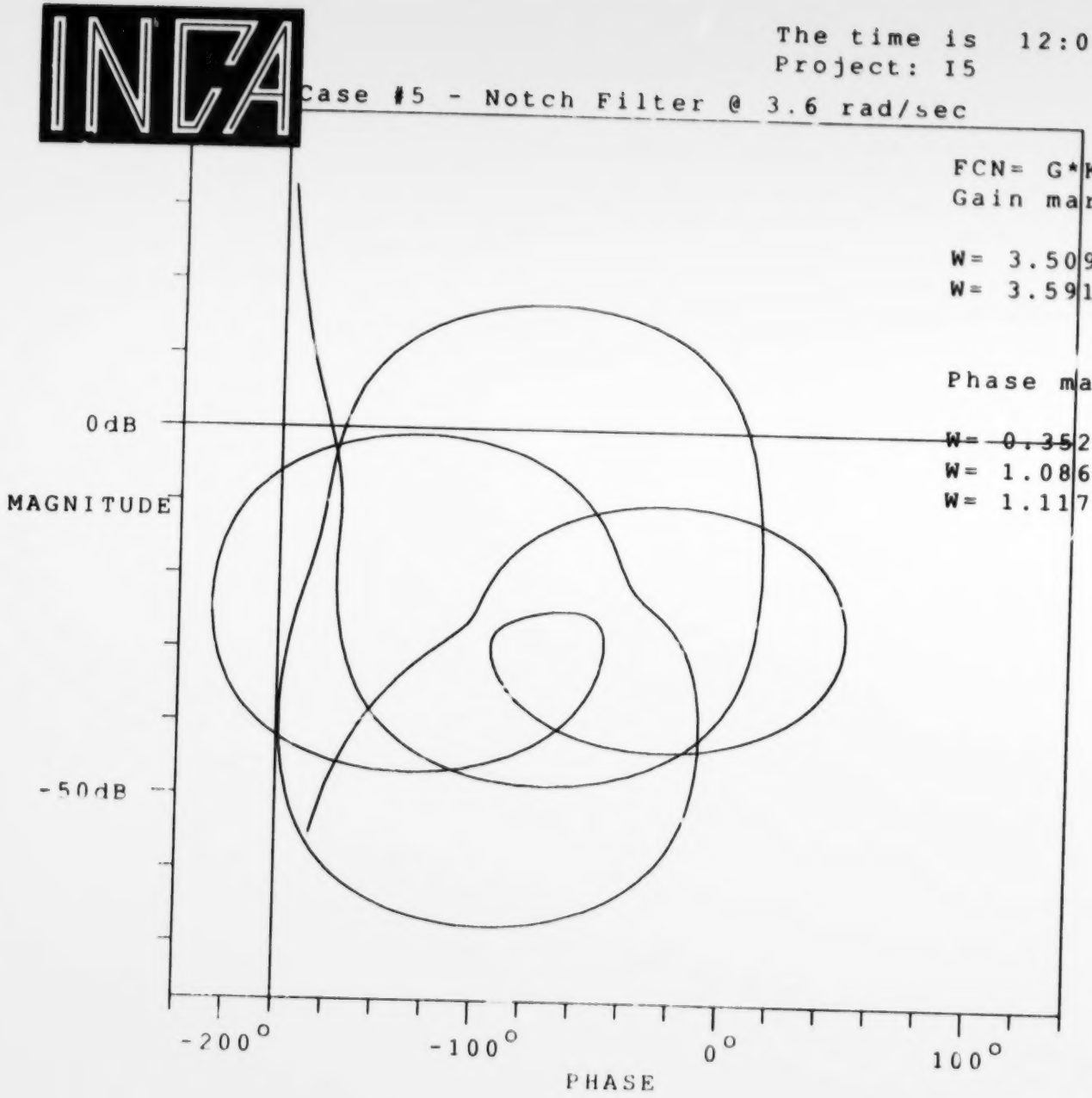
Origin=-88.45dB


INCA
The time is 11:52:15.96 3-JUL-1988
Project: I5
Case #4 - Notch Filter @ 3.4 rad/sec
0
dB
-50
.1
1
10
100°
0°
PHASE
-100°
-200°
.1
1
10
frequency (rad/sec)

The time is 11:56:00.56 3-JUL-1988
Project: I5

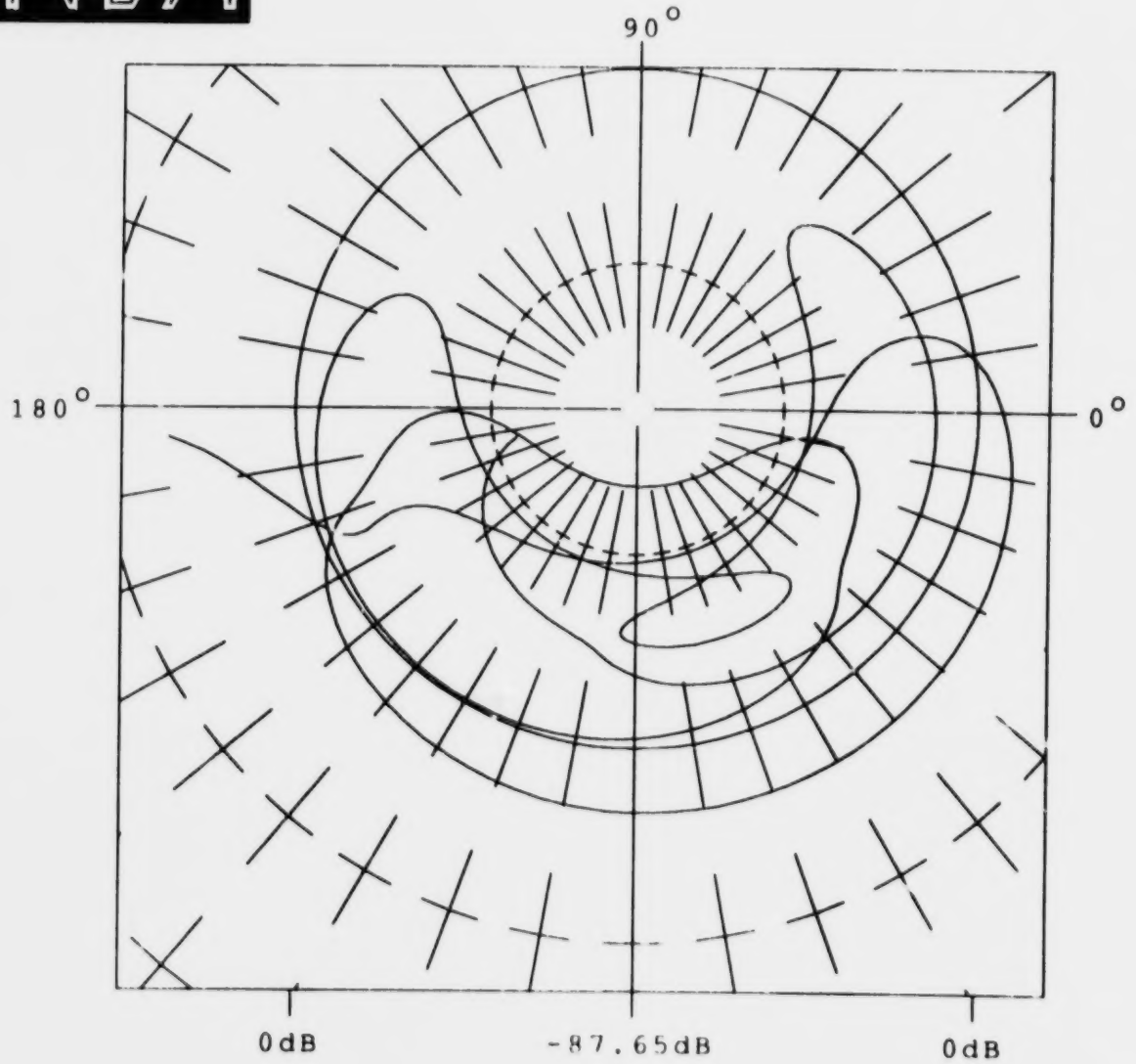



The time is 10:26:49.70 3-JUL-1988
Project: I5
Case #5 - Notch Filter @ 3.6 rad/sec
IMAG
6
4
2
0
G= 2.0766
G/I= 1.000000
-.20
-.15
-.10
-.05
-0
REAL

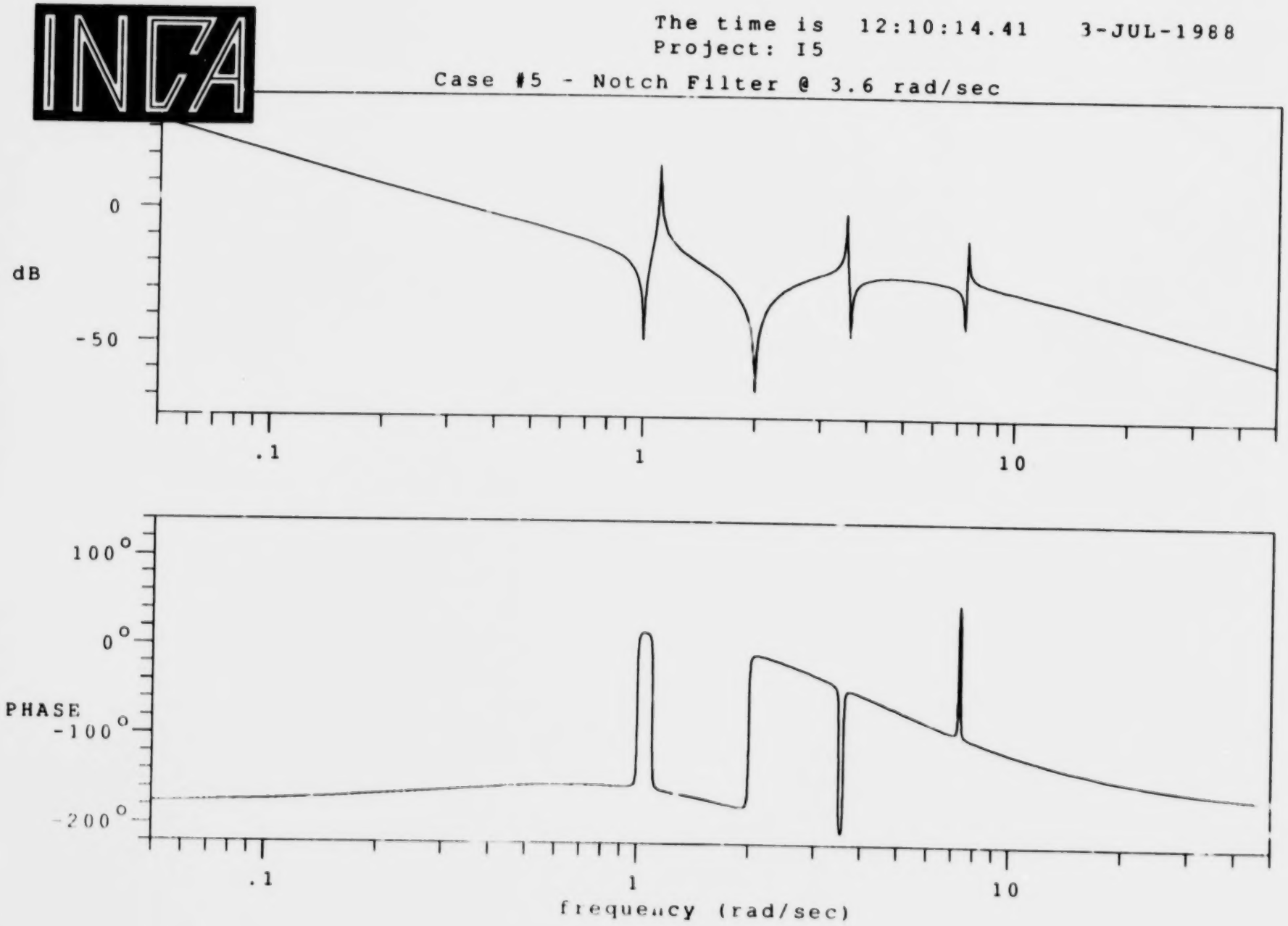

INCA
The time is 12:01:34.33 3-JUL-1988
Project: I5
Case #5 - Notch Filter @ 3.6 rad/sec
FCN= G*K5
Gain margins (P= 180)
W= 3.5092 rps M=-6.1049 dB
W= 3.5912 rps M=-42.035 dB
Phase margins (M= 0dB)
W= 0.3527 rps P=-160.16
W= 1.0865 rps P= 9.0433
W= 1.1175 rps P=-154.67
0dB
MAGNITUDE
-50dB
-200°
-100°
0°
100°
PHASE

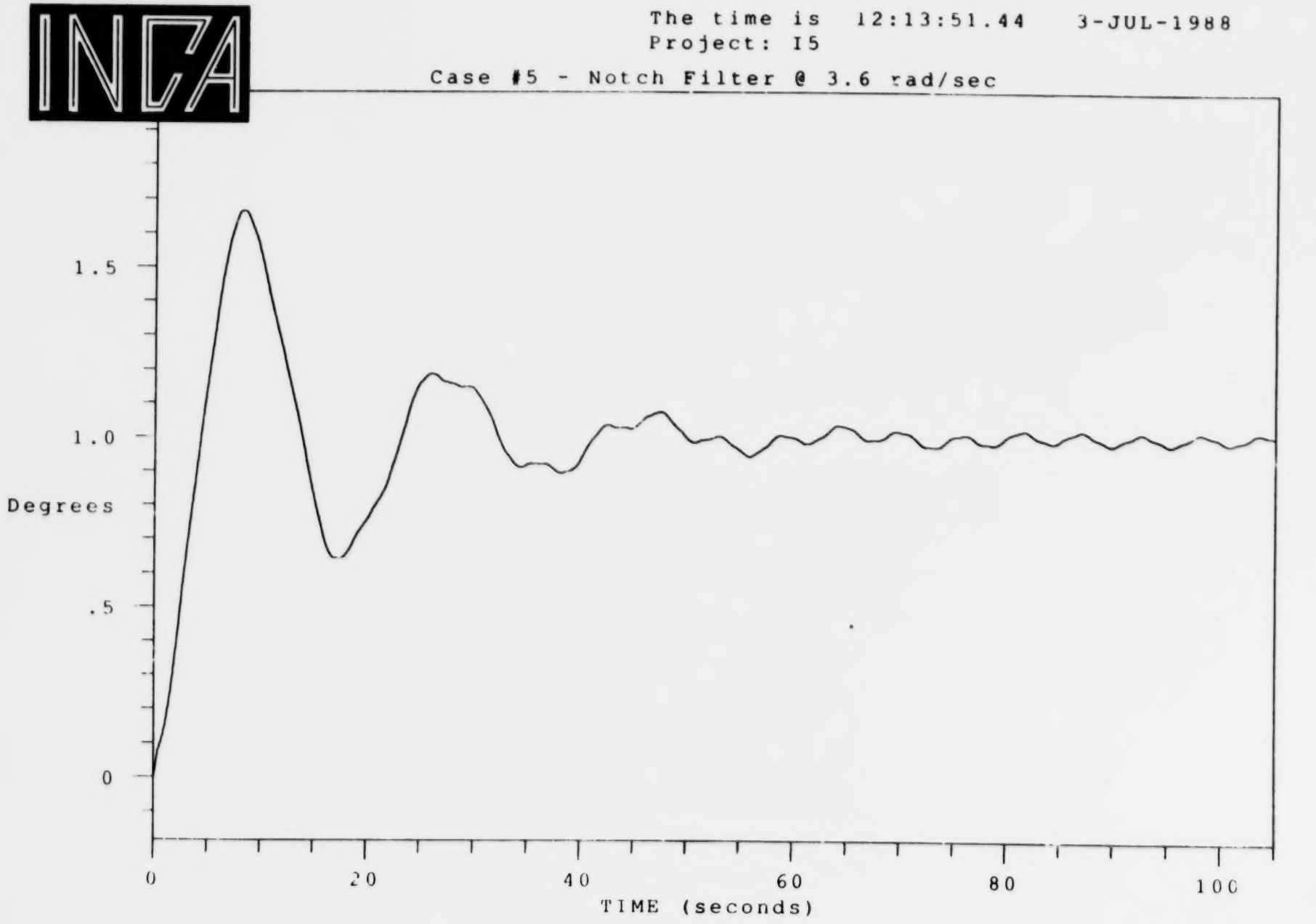
The time is 12:05:00.16 3-JUL-1988
Project: I5

se #5 - Notch Filter @ 3.6 rad/sec



Origin=-87.65dB


INCA
The time is 12:10:14.41 3-JUL-1988
Project: I5
Case #5 - Notch Filter @ 3.6 rad/sec
0
dB
-50
.1
1
10
100°
0°
PHASE
-100°
-200°
.1
1
10
frequency (rad/sec)


INCA
The time is 12:13:51.44 3-JUL-1988
Project: I5
Case #5 - Notch Filter @ 3.6 rad/sec
1.5
1.0
Degrees
.5
0
0
20
40
60
80
100
TIME (seconds)